DEFINITION---The graph of the ordered pairs (x, y) where: $x = f(t)$, $y = g(t)$ are functions defined on an interval I of t-values is a PARAMETRIC CURVE. The equations are PARAMETRIC EQUATIONS for the curve, the variable t is a PARAMETER, and I is the PARAMETER INTERVAL.

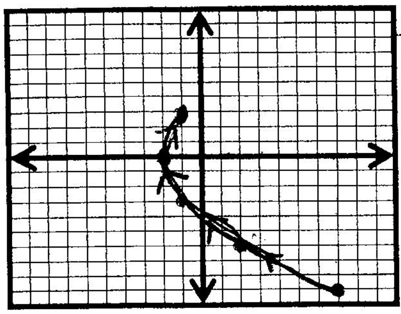
*When we give parametric equations and a parameter interval for a curve, we have "**parametrized**" the curve.

A "parametrization**" of a curve consists of the _parametric equations_ AND the _interval_ of t-values.

EXAMPLE 1 Graphing Parametric Equations

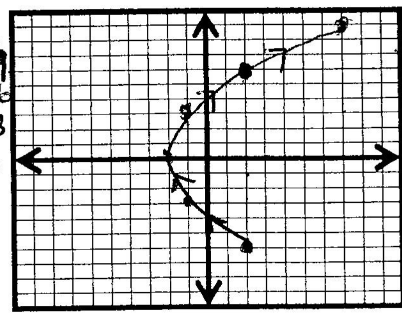
For the given parameter interval, graph the parametric equations using your graphing calculator & note the difference you see: $x = t^2 - 2$, $y = 3t$

(a) $-3 \le t \le 1$



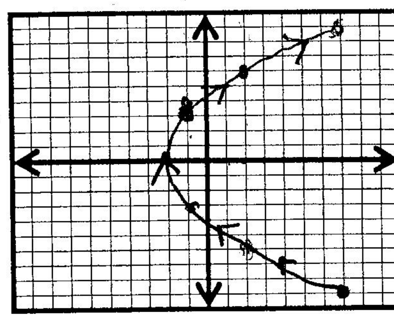
t	x	y
-3	7	-9
-2	2	-6
-1	-1	-3
0	-2	0
1	-1	3

(b) $-2 \le t \le 3$



t	x	y
-2	2	-6
-1	-1	-3
0	-2	0
1	-1	3
2	2	6
3	7	9

(c) $-3 \le t \le 3$



t	x
-3	7
-2	2
-1	-1
0	-2
1	-1
2	2
3	7

*What do you think is the point of doing this example?

If we do not specify a parameter interval for the parametric equations $x = f(t)$, $y = g(t)$, it is understood that the parameter t can take on all values which produce real numbers for x and y.

EXAMPLE 2 Eliminating the Parameter

Eliminate the parameter and identify the graph of the parametric curve $x = 1 - 2t$, $y = 2 - t$, $-\infty \le t \le \infty$.
(without calculator)

$x - 1 = -2t$

$\frac{x-1}{-2} = t$

$y = 2 - \left(\frac{x-1}{-2}\right)$

$y = 2 + \frac{1}{2}x - \frac{1}{2}$

$y = \frac{1}{2}x + \frac{3}{2}$

line $m = \frac{1}{2}$ $b = \frac{3}{2}$

EXAMPLE 3 Eliminating the Parameter

Eliminate the parameter and identify the graph of the parametric curve : $x = t^2 - 2$, $y = 3t$.
(without a calculator)

$t = \frac{y}{3}$

$x = \left(\frac{y}{3}\right)^2 - 2$

$x = \frac{1}{9}y^2 - 2$

$x + 2 = \frac{1}{9}y^2$ parabola opens right

EXAMPLE 4 Eliminating the Parameter

Eliminate the parameter and identify the graph of the parametric curve: $x = 2\cos t$, $y = 2\sin t$, $0 \le t \le 2\pi$

we know $\sin^2\theta + \cos^2\theta = 1$

$\frac{x}{2} = \cos t$ $\quad \frac{y}{2} = \sin t$

$\left(\frac{y}{2}\right)^2 + \left(\frac{x}{2}\right)^2 = 1$

$\frac{y^2}{4} + \frac{x^2}{4} = 1$

$y^2 + x^2 = 4.$

circle
center (0,0)
r = 2

EXAMPLE 5 Finding Parametric Equations for a Line

Find a parametrization of the line through the points (-2, 3) & (3, 6).

start

$\text{Slope} = \frac{6-3}{3 - -2} = \frac{3 \text{ rise}}{5 \text{ run}}$

$\langle -2, 3 \rangle + t\langle 5, 3 \rangle$ (horiz, vert)

$\langle -2 + 5t, 3 + 3t \rangle$

$x = -2 + 5t$
$y = 3 + 3t$

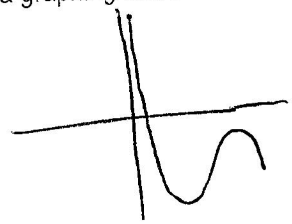
12

EXAMPLE 6 Simulating Horizontal Motion
Gary walks along a horizontal line (think of it as a number line) with the coordinate of his position (in meters) given by $s = -0.1(t3 - 20t2 + 110t - 85)$ where $0 \le t \le 12$. Use parametric equations and a graphing calculator to simulate his motion. Estimate the times when Gary changes direction.

$x = \text{time} = t$

$y = \text{position} = -.1\,(t^3 - 20t^2 + 110t - 85)$

about $t = 4$ and $t = 9.5$



EXAMPLE 7 Finding Height of a Projectile
A projectile is launched straight up from ground level $s_0 = 0$ with an initial velocity of 288 ft /sec.

$s(t) = -16t^2 + 288t$

Projectile Motion

Suppose an object is launched vertically from a point s_0 feet above the ground with an initial velocity of v_0 feet per second. The vertical position s (in feet) of the object t seconds after it is launched is

$$s = -16t^2 + v_0 t + s_0.$$

(a) When is the projectile's height above ground 1152 ft?

$-16t^2 + 288t = 1152$ $\quad t = 6\text{ sec}$ and $t = 12\text{ sec}$

(b) When is the projectile's height above ground at least 1152 ft?

$6 \le t \le 12$

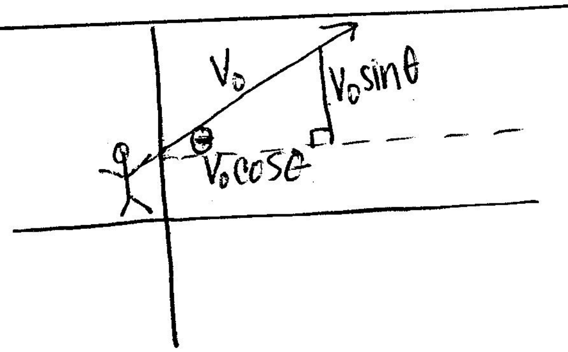
EXAMPLE 8 Simulating Projectile Motion
A distress flare is shot straight up from a ship's bridge 75 ft above the water with an initial velocity of 76 ft/sec. Graph the flare's height against time, give the height of the flare above water at each time, and simulate the flare's motion for each length of time.

(a) 1 sec (b) 2 sec (c) 4 sec (d) 5 sec

$s(t) = -16t^2 + 76t + 75$

135 ft 163 ft 123 ft 55 ft

Suppose that a baseball is thrown from a point y_0 feet above ground level with an initial speed of v_0 ft/sec at an angle θ with the horizontal. The initial velocity can be represented by the vector $\mathbf{v} = \langle v_0 \cos\theta, v_0 \sin_0\theta\rangle$.
The path of the object is modeled by the parametric equations:



x = $V_0 \cos\theta \cdot t$
[The x-component is simply distance = (x-component of initial velocity) × time]

and

y = $-16t^2 + V_0 \sin\theta\, t + y_0$
[The y-component is the familiar vertical projectile-motion equation using the y-component of initial velocity vector]

EXAMPLE 9 Hitting a Baseball
Kevin hits a baseball at 3 ft above the ground with an initial speed of 150 ft/sec at an angle of 18° with the horizontal. Will the ball clear a 20-ft wall that is 400 ft away?

$x = 150 \cos 18^\circ \cdot t$

$y = -16t^2 + 150 \sin 18^\circ t + 3$

$400 = 150 \cos 18^\circ t$

$t = 2.804\text{ sec}$

$y(2.80) = 7.178\text{ ft}$, hits ground doesn't clear the wall