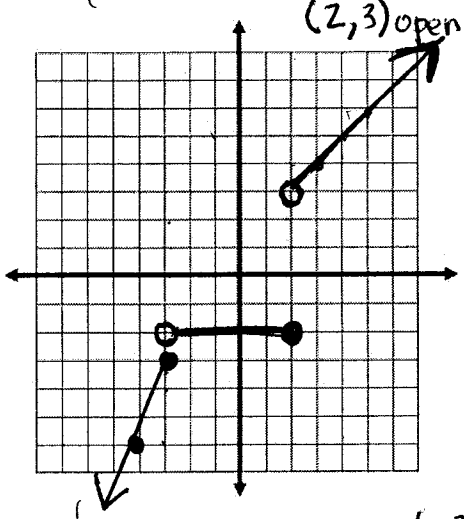
Graphing Piecewise Functions

Example 1 Graph each piecewise function:

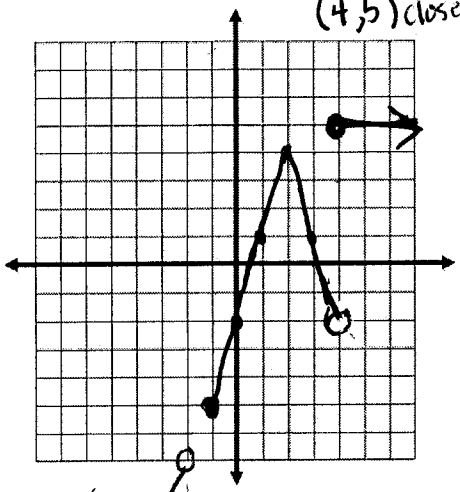
A. $y=\begin{cases} 3x+6 & \text{for } x \le -3 \\ -2 & \text{for } -3<x\le 2 \\ x+1 & \text{for } x>2 \end{cases}$

(-3,-3) closed
(-3,-2) open
(2,-2) closed
(2,3) open



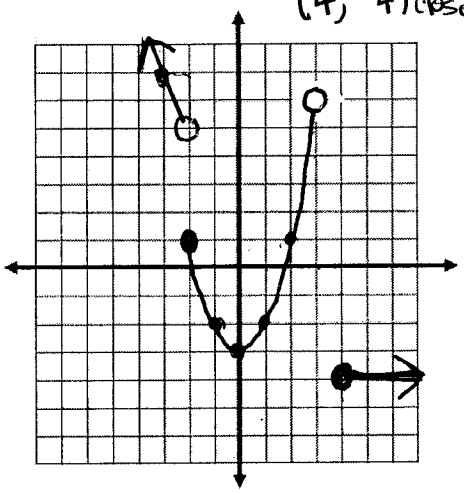
B. $y=\begin{cases} 3x-1 & \text{for } x<-2 \\ -3|x-2|+4 & \text{for } -1\le x<4 \\ 5 & \text{for } x\ge 4 \end{cases}$

(-2,-7) open
(-1,-5) closed
(4,-2) open
(4,5) closed



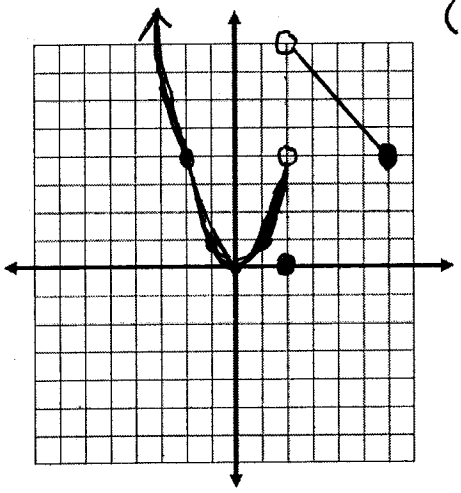
C. $y=\begin{cases} -2x+1 & \text{for } x<-2 \\ x^2-3 & \text{for } -2\le x<3 \\ -4 & \text{for } x\ge 4 \end{cases}$

(-2,5) open
(-2,1) closed
(3,8) open
(4,-4) closed



D. $y=\begin{cases} x^2 & \text{for } x<2 \\ 0 & \text{for } x=2 \\ 10-x & \text{for } 2<x\le 6 \end{cases}$

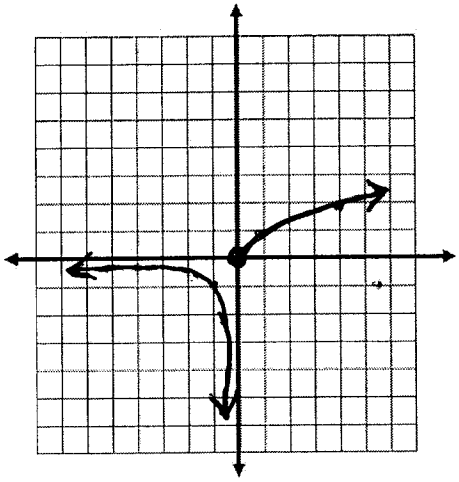
(2,4) open
(2,0) closed
(2,8) open
(6,4) closed



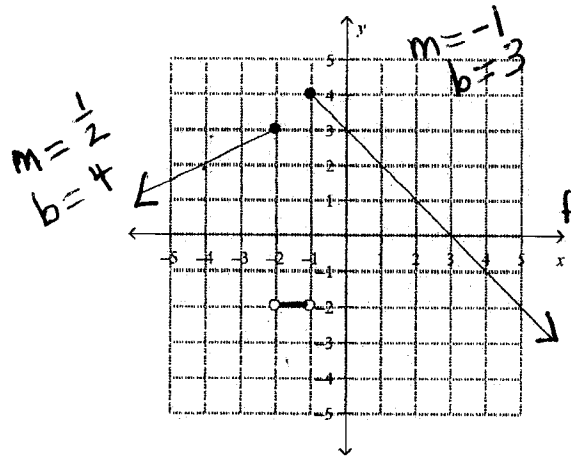
Example 2 Graph the piecewise function:

$$f(x)=\begin{cases} \dfrac{1}{x} & \text{if } x<0 \\ \sqrt{x} & \text{if } x\ge 0 \end{cases}$$

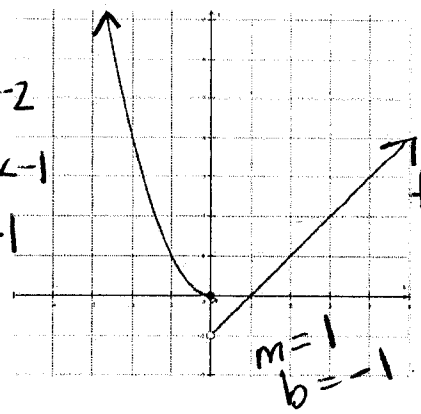
(0,0) closed



Example 3 Given a graph, find a piecewise function that describes it:



$$f(x)=\begin{cases}\frac{1}{2}x+4, & x\le -2\\ -2, & -2<x<-1\\ -x+3, & x\ge -1\end{cases}$$



$$f(x)=\begin{cases}x^2, & x\le 0\\ x-1, & x>0\end{cases}$$

Applications of Piecewise Functions

Example 4 In May 2012, Powell Gas had the following rate schedule for natural gas usage in single-family residences:

Monthly Customer Charge $6.45
Distribution Charge
- 1st 20 therms $0.2012/therm
- Next 30 therms $0.1117/therm
- Over 50 therms $0.0374/therm
- Gas supply charge $0.7268/therm

A. What is the charge for using 40 therms?
B. What is the charge for using 202 therms?
C. Construct a function that gives the monthly charge C for x therms of gas.

A. $6.45+20(.2012)+20(.1117)+40(.7268)=\$41.78$

B. $6.45+20(.2012)+30(.1117)+152(.0374)+202(.7268)=\$166.32$

C.
$$C(x)=\begin{cases}6.45+x(.2012)+x(.7268), & \\ \underline{6.45}+\underline{20(.2012)}+(x-20)(.1117)+x(.7268) & \\ 6.45+20(.2012)+30(.1117)+(x-50)(.0374)+x(.7268) & \end{cases}$$

$$C(x)=\begin{cases}.928x+6.45, & 0\le x\le 20\\ .8385x+8.24, & 20<x\le 50\\ .7642x+11.955, & x>50\end{cases}$$

Example 5 An economy car costs \$95 per week. Extra days cost \$24 per day until the rate exceeds the weekly rate, in which case the weekly rate applies. Find the cost C of renting an economy car as a piecewise-defined function of the number x of days used, where $7 \le x \le 14$. Graph the function.

$C(x)$ cost of renting a car

$x =$ # of days

days x	cost $C(x)$
7	95
8	119
9	143
10	167
11	190
12	190
13	190
14	190

$$C(x) = \begin{cases} 95 + 24(x-7), & 7 \le x \le 10 \\ 190, & 11 \le x \le 14 \end{cases}$$