Pre-Calculus Name: ______________________________

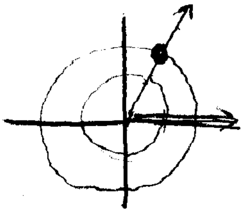
Notes—(6.4) Polar Coordinates

A **polar coordinate system** is a plane with a point *O*, the **pole**, and a ray from *O*, the **polar axis**. Each point *P* in the plane is assigned as **polar coordinates** as follows: *r* is the **directed distance** from *O* to *P* and θ is the **directed angle** whose initial side is on the polar axis and whose terminal side is on the line *OP*.

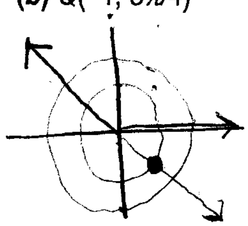
As in trigonometry, we measure θ as positive when moving counterclockwise and negative when moving clockwise. If $r > 0$, then *P* is on the terminal side of θ. If $r < 0$, then *P* is on the terminal side of $\theta + \pi$. We can use radian or degree measure for the angle θ.

EXAMPLE 1 Plotting Points in the Polar Coordinate System
Plot the points with the given polar coordinates.

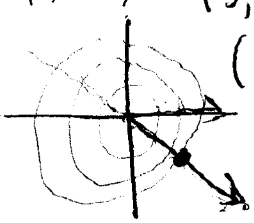
(a) $P(2, \pi/3)$ $\left(2, \frac{7\pi}{3}\right)$ $(2, -300^\circ)$



(b) $Q(-1, 3\pi/4)$ $\left(1, \frac{-\pi}{4}\right)$ $(-1, -225^\circ)$



(c) $R(3, -45°)$ $(3, 315^\circ)$ $(-3, 135^\circ)$

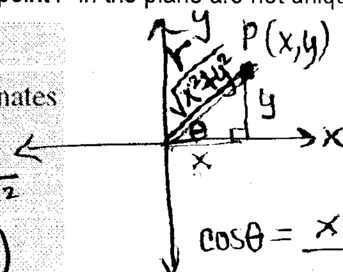


NOTE: Each polar coordinate pair determines a unique point. However, the polar coordinates of a point *P* in the plane are not unique.

Coordinate Conversion Equations

Let the point P have polar coordinates (r, θ) and rectangular coordinates (x, y). Then

$$x = r\cos\theta, \qquad r^2 = x^2 + y^2 \rightarrow r = \sqrt{x^2+y^2}$$
$$y = r\sin\theta, \qquad \tan\theta = \frac{y}{x} \quad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$



$\cos\theta = \frac{x}{r}$

$x = r\cos\theta$

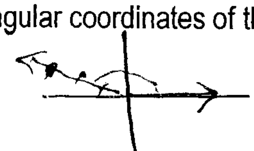
$\sin\theta = \frac{y}{r}$

$y = r\sin\theta$

✱ be mindful of the quadrant

EXAMPLE 2 Converting from Polar to Rectangular Coordinates
Find the rectangular coordinates of the points with the given polar coordinates.

(a) $P(3, 5\pi/6)$ (↓ r, ↓ θ)

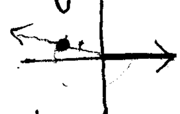


$x = r\cos\theta = 3\cos\frac{5\pi}{6} = 3\left(-\frac{\sqrt{3}}{2}\right) = -\frac{3\sqrt{3}}{2}$

$y = r\sin\theta = 3\sin\frac{5\pi}{6} = 3\left(\frac{1}{2}\right) = \frac{3}{2}$

$\left(-\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$

(b) $Q(2, -200°)$



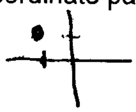
$x = 2\cos(-200^\circ)$

$y = 2\sin(-200^\circ)$

$(-1.88, .68)$

EXAMPLE 3 Converting from Rectangular to Polar Coordinates
Find two polar coordinate pairs for the points with given rectangular coordinates.

(a) $P(-1, 1)$ (x y)



2nd quad

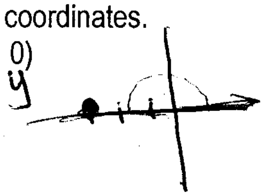
$r = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$

$\tan\theta = \frac{1}{-1} = -1$

$\theta = \tan^{-1}(-1)$
135°

$(\sqrt{2}, 135^\circ)$
$(-\sqrt{2}, -45^\circ)$
$(-\sqrt{2}, 315^\circ)$

(b) $Q(-3, 0)$ (x y)



$r = \sqrt{(-3)^2 + (0)^2} = 3$

$\tan\theta = \frac{0}{-3}$

$\theta = \tan^{-1}(0)$

$(3, 180^\circ)$
$(-3, 0^\circ)$

Equation Conversion

We can use the Coordinate Conversion Equations to convert polar form to rectangular form and vice versa. For example, the polar equation $r = 4\cos\theta$ can be converted to rectangular form as follows:

Just as with parametric equations, the domain of a polar equation in r and θ is understood to be all values of θ for which the corresponding values of r are real numbers. You must also select a value for θmin and θmax to graph in polar mode.

Converting from Polar Form to Rectangular Form: Convert the polar equation to rectangular form ~~and identify the graph~~

Ex4) $r = 4\sec\theta$

$$\frac{r}{1} = \frac{4}{\cos\theta}$$

$$r\cos\theta = 4$$

$$x = 4$$

vertical line!

Ex5) $r = 4\cos\theta$

$$r(r) = 4(r)\cos\theta$$

$$r^2$$

$$\boxed{x^2+y^2=4x}$$

ellipse

E6) $r = 3\cos\theta$

$$r^2 = 3r\cos\theta$$

$$\boxed{x^2+y^2=3x}$$

Ex7) $r^2 = -3\sec\theta$

$$\frac{r^2}{r} = \frac{-3}{r\cos\theta}$$

$$r = \frac{-3}{x}$$

$$\boxed{\sqrt{x^2+y^2} = -\frac{3}{x}}$$

Ex8) $r\left(\frac{r}{3\tan\theta}\right) = (\sin\theta)\, r$

$$\frac{r^2}{3\tan\theta} = r\sin\theta$$

$$\frac{x^2+y^2}{3\cdot\frac{y}{x}} = \frac{y}{1}$$

$$x^2+y^2 = \frac{3y^2}{x}$$

$$\boxed{x^3+xy^2=3y^2}$$

Converting from Rectangular Form to Polar Form: Convert from rectangular form to polar form.

Ex8) $x^2 + y^2 = 1$

Ex9) $y = 2x+1$

Ex10) $y = \frac{3}{x}$

Ex11) $(x-3)^2 + (y-2)^2 = 13$