

Equation Conversion

We can use the Coordinate Conversion Equations to convert polar form to rectangular form and vice versa. For example, the polar equation $r = 4 \cos \theta$ can be converted to rectangular form as follows:

Just as with parametric equations, the domain of a polar equation in r and θ is understood to be all values of θ for which the corresponding values of r are real numbers. You must also select a value for θ_{\min} and θ_{\max} to graph in polar mode.

Converting from Polar Form to Rectangular Form: Convert the polar equation to rectangular form and identify the graph.

Ex4) $r = 4 \sec \theta$

$$\frac{r}{1} = \frac{4}{\cos \theta}$$

$$r \cos \theta = 4$$

$x = 4$
vertical line!

Ex5) $r = 4 \cos \theta$

$$r(r) = 4(r \cos \theta)$$

$$r^2 = 4x$$

ellipse

E6) $r = 3 \cos \theta$

$$r^2 = 3 r \cos \theta$$

$$x^2 + y^2 = 3x$$

Ex7) $r^2 = -3 \sec \theta$

$$\frac{r^2}{r} = \frac{-3}{r \cos \theta}$$

$$r = \frac{-3}{x}$$

$$\sqrt{x^2 + y^2} = -\frac{3}{x}$$

Ex8) $\left(\frac{r}{3 \tan \theta}\right) = (\sin \theta) r$

$$\frac{r^2}{3 \tan \theta} = r \sin \theta$$

$$\frac{x^2 + y^2}{3 \cdot \frac{y}{x}} = \frac{y}{1}$$

$$x^2 + y^2 = \frac{3y^2}{x}$$

$$x^3 + xy^2 = 3y^2$$

Converting from Rectangular Form to Polar Form: Convert from rectangular form to polar form.

Ex8) $x^2 + y^2 = 1$

$$r^2 = 1$$

$$r = \pm 1$$

Ex9) $y = 2x + 1$

$$r \sin \theta = 2r \cos \theta + 1$$

$$r \sin \theta - 2r \cos \theta = 1$$

$$r(\sin \theta - 2 \cos \theta) = 1$$

$$r = \frac{1}{\sin \theta - 2 \cos \theta}$$

Ex10) $y = \frac{3}{x}$

$$r \sin \theta = \frac{3}{r \cos \theta}$$

$$r^2 \sin \theta \cos \theta = 3$$

$$r^2 = \frac{3}{\sin \theta \cos \theta}$$

$$r = \pm \sqrt{\frac{3}{\sin \theta \cos \theta}}$$

Ex11) $(x-3)^2 + (y-2)^2 = 13$

$$x^2 - 6x + 9 + y^2 - 4y + 4 = 13$$

$$r^2 - 6r \cos \theta - 4r \sin \theta + 13 = 13$$

$$r^2 - 6r \cos \theta - 4r \sin \theta = 0$$

$$r(r - 6 \cos \theta - 4 \sin \theta) = 0$$

$r = 0$

single point

(at the pole)

$$r - 6 \cos \theta - 4 \sin \theta = 0$$

$$r = 6 \cos \theta + 4 \sin \theta$$