

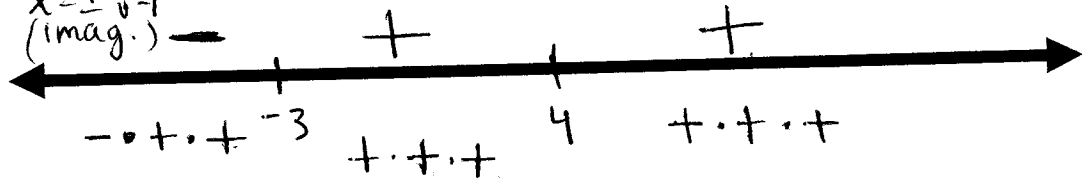
2.9---SOLVING INEQUALITIES IN ONE VARIABLE

RECALL: With inequalities a solution is ANY VALUE that will make the statement true.

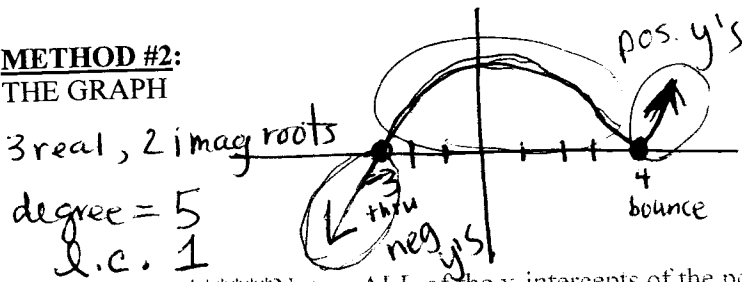
Ex1) Given the function below, determine the values of x that cause to be:

$f(x) = (x+3)(x^2+1)(x-4)^2$ (a) zero (b) positive (c) negative
 find real zeros $x^2+1=0$ $x=-3, x=4$ $(-3, 4) \cup (4, \infty)$ $(-\infty, -3)$
 $x=-3, x=4$ $x^2=-1$ $x = \pm \sqrt{-1}$ (imag.)

METHOD #1:
THE SIGN CHART



METHOD #2:
THE GRAPH



*****Note---ALL of the x-intercepts of the polynomial are placed on the sign chart*****
 Using the sign chart above we have enough information to find the solutions to each of the 4 following inequalities:

$(x+3)(x^2+1)(x-4)^2 > 0$ pos
 $(-3, 4) \cup (4, \infty)$
 $(x+3)(x^2+1)(x-4)^2 \geq 0$ pos & zero
 $[-3, \infty)$

$(x+3)(x^2+1)(x-4)^2 < 0$ neg
 $(-\infty, -3)$
 $(x+3)(x^2+1)(x-4)^2 \leq 0$ neg & zero
 $(-\infty, -3] \cup [4, 4]$

Ex2) Solve

(a) $(x^2+7)(3x^2+1) \geq 0$ (pos, zero) (b) $(x^2+7)(3x^2+1) \leq 0$ (neg, zero)

$x^2+7=0$ $3x^2+1=0$
 $x^2=-7$ $x^2=-\frac{1}{3}$
 $x = \pm \sqrt{-7}$ $x = \pm \sqrt{-\frac{1}{3}}$
 (imag) (imag)
 no real zeros

test some value

$x=0: + \cdot + = +$

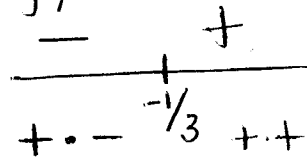
$(-\infty, \infty)$

Ex3) Solve:

(a) $(x^2+x+7)(3x+1) \geq 0$ (pos, zero) (b) $(x^2+x+7)(3x+1) > 0$ (pos)

quad. formula $3x+1=0$
 $x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(7)}}{2(1)}$ $x = -\frac{1}{3}$

$= \frac{-1 \pm \sqrt{-27}}{2}$
 (imag.)



$[-\frac{1}{3}, \infty)$

$$\text{Ex 4) } \frac{(3x-1)(2x+5)}{(2x+5)(3x-1)} = \frac{(5x^2+18)}{(3x-1)(2x+5)}$$

$$\text{Ex 5) } \frac{(t+4)(t-4)}{t(t-4)} = \frac{(-16)}{t(t-4)}$$

$$\text{LCD} = (3x-1)(2x+5)$$

$$\text{LCD} = t(t-4)$$

$$(3x-1)(2x+2) - (2x+5)(x-4) = 5x^2 + 18$$

$$(t-4)(t+4) + -4t = -16$$

$$\underline{6x^2 + 6x - 2x - 2} - \underline{2x^2 + 8x - 5x + 20} = 5x^2 + 18 \quad t^2 - 16 - 4t = -16$$

$$4x^2 + 7x + 18 = 5x^2 + 18$$

$$t^2 - 4t = 0$$

$$-x^2 + 7x = 0$$

$$\begin{matrix} -x=0 & x-7=0 \\ \boxed{x=0} & \boxed{x=7} \end{matrix}$$

$$-x(x-7) = 0$$

$$t(t-4) = 0$$

Now You Try:

$$6) \frac{5x}{x-2} = 7 + \frac{10}{x-2}$$

$$\frac{(x+1)(x-1)}{(x+1)(x-1)} = \frac{(12 + \frac{3}{1})}{(x+1)(x-1)}$$

~~t=0~~ ~~t-4=0~~
~~t=4~~
no solution

$$\text{LCD} = (x+1)(x-1)$$

$$(x-1)(4x+1) = 12 + 3(x+1)(x-1)$$

$$4x^2 + x - 4x - 1 = 12 + 3x^2 - 3$$

$$x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

$$x-5=0 \quad x+2=0$$

$$\boxed{x=5} \quad \boxed{x=-2}$$

$$8) \frac{1}{x-2} + \frac{x-3}{7-x} = \frac{x+1}{-x^2+9x-14}$$