Precalculus Unit 2 Notes—Polynomial Functions

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*******Graphing Polynomial Functions*******

Not only are graphs of polynomials unbroken without jumps or holes, but they are *smooth* lines or curves, with no sharp corners or cusps.

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Investigation 1 - End Behavior:

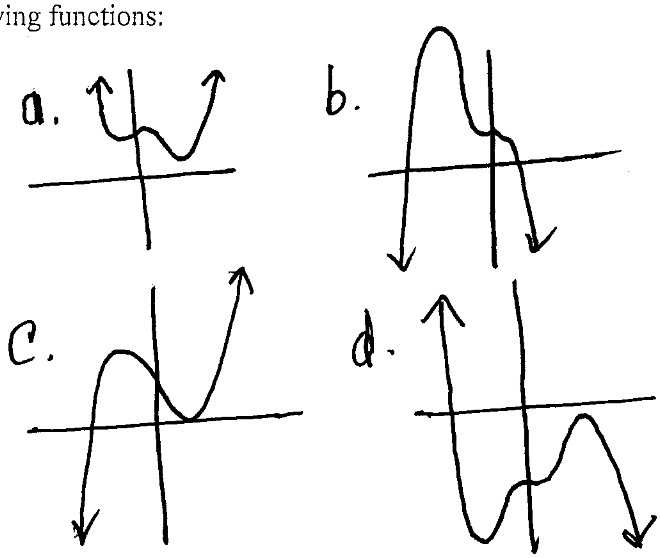
1. Using a graphing calculator, sketch the following functions:

a. $y = 3x^4 - 7x^3 + x^2 + 9$

b. $y = -1/2x^6 - 4x^5 + 2x^3 - 11x + 5$

c. $y = 2x^3 + 5x^2 - 3x + 1$

d. $y = -3x^5 + 7x^3 - 5$



2. What affects the right end behavior?

leading coefficient

$+ \Rightarrow \infty$, $- \Rightarrow -\infty$

3. What affects the left end behavior?

degree "n"

even ⇒ LEB = REB , odd ⇒ LEB opposite of REB

THEOREM ----**A polynomial function of degree** __n__ **has at most** __n−1__ **local extrema and at most** __n__ **zeros.**

End Behavior of Polynomial Functions

In order to determine the end behavior of a polynomial function you need only 2 pieces of information:

1st: You must know the __degree__ of the polynomial. If the __degree__ is __even__ the LEFT END BEHAVIOR (L.E.B) & the RIGHT END BEHAIVOR (R.E.B) will be ~~the~~ Same. If the __degree__ is __odd__ then the L.E.B. and the R.E.B. will be __opposite__.

2nd: You must know the sign of the __leading coefficient__ (L.C.) of the polynomial. If the __L.C__ is POSITIVE then the R.E.B. will be: $\lim_{x\to\infty} f(x) = \infty$. However, if the __L.C__ is NEGATIVE then the R.E.B. will be: $\lim_{x\to\infty} f(x) = -\infty$.

Zeros of Polynomial Functions

Recall that finding the real-number zeros of a function f is equivalent to finding the x-intercepts of the graph of $y = f(x)$ or the solutions to the equation $f(x) = 0$.

Investigation 2 - Local Extrema and Zeros

1. Using a graphing calculator, investigate various third degree polynomial functions to see how many extrema and how many zeros the function can have.

at most 2   at most 3

2. Using a graphing calculator, investigate various fourth degree polynomial functions to see how many extrema and how many zeros the function can have.

at most 3   at most 4

⇨ **In general, if your degree is "n", how many extrema are possible?** n−1

⇨ **In general, if your degree is "n", how many zeros are possible?** n

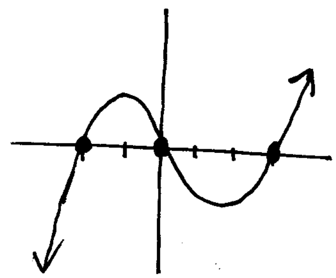
Example 1   Find the zeros of $f(x) = x^3 - x^2 - 6x$ and then sketch the graph using your knowledge of intercepts and end behavior.

$$x^3 - x^2 - 6x = 0$$
$$x(x^2 - x - 6) = 0$$
$$x(x-3)(x+2) = 0$$
$$x = 0 \quad x - 3 = 0 \quad x + 2 = 0$$

$x = 0, 3, -2$

LEB $\lim_{x\to-\infty} f(x)$

REB $\lim_{x\to\infty} f(x)$



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***** Multiplicity of a Zero of a Polynomial Function*****

If f is a polynomial function & $(x - c)^m$ is a factor of f then c is a zero of **multiplicity** m of f.

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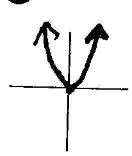
Zeros of Odd and Even Multiplicity

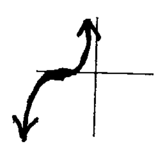
If a polynomial function f has a real zero c of odd multiplicity, then the graph of f crosses the x-axis at $(c, 0)$ and the value of f changes sign at $x = c$. If a polynomial function f has a real zero c of even multiplicity, then the graph of f does not cross the x-axis at $(c, 0)$ and the value of f does not change sign at $x = c$.

⇨ If the multiplicity of a zero is 1, it will cross the x-axis in the typical "straight through" manner.

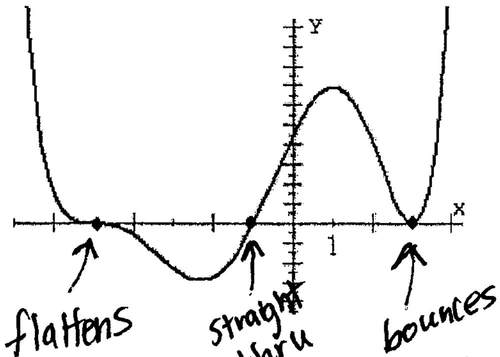
⇨ If the multiplicity of a zero is EVEN, it will bounce at the x-axis & will NOT cross through.



⇨ If the multiplicity of a zero is greater than 1 & ODD, it will flatten out at the x-axis & will cross through.



Example 2 Identify the zeros of the polynomial function:



Multiplicity of 1: $x = -1$

Even Multiplicity: $x = 3$

Odd Multiplicity > 1: $x = -5$

could be $f(x) = (x+1)(x-3)^2(x+5)^3$

Example 3 State the degree and find the zeros of each function. State the multiplicity of each zero and whether the graph crosses the x-axis at the corresponding x-intercept. Then sketch the graph of each polynomial.

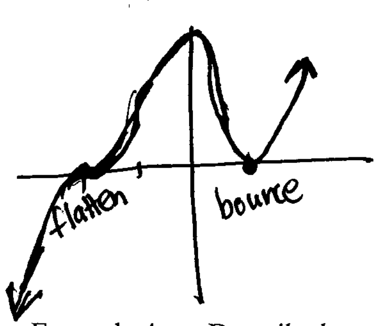
(a) $f(x) = (x + 2)^3(x - 1)^2$

degree 5

$(x+2)^3 = 0$ $(x-1)^2 = 0$

$x+2 = 0$ $x - 1 = 0$

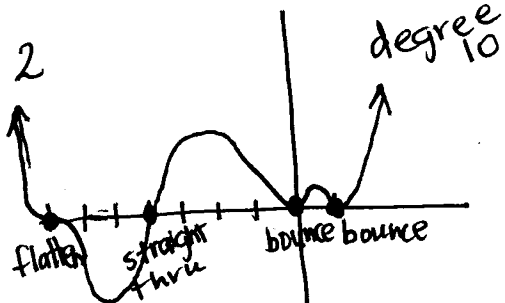
$x = -2$ mult of 3 $x = 1$ mult. of 2



(b) $f(x) = x^2(x + 7)^3(x - 1)^4(x + 4)^1$

$x = 0$ mult 2 $x = -7$ mult 3 $x = 1$ mult 4 $x = -4$ mult 1

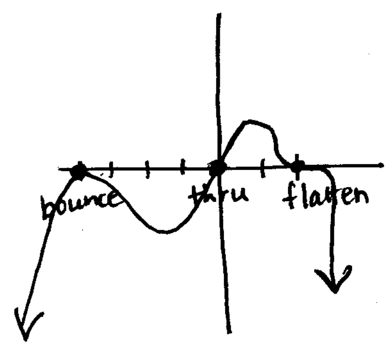
degree 10



(c) $f(x) = -x^1(x + 4)^2(x - 2)^3$

$x = 0$ mult 1 $x = -4$ mult 2 $x = 2$ mult 3

degree 6



Example 4 Describe how to transform the graph of a basic monomial function into the graph of the given polynomial.

a) $f(x) = 4(x+1)^3$

$y = x^3$
left 1
vertical stretch *4

b) $g(x) = -(x-2)^4 + 5$

$y = x^4$
right 2
reflect over x-axis
shift up 5

c) $h(x) = 2(x-3)^5 - 1$

$y = x^5$
right 3
vertical stretch * 2
shift down 1