Real Zeros of Polynomials (2.4)

Remember long division? dividend How about with polynomials?

divisor

Ex1) $\dfrac{1373}{3}$

$$\begin{array}{r} 457\tfrac{2}{3} \\ 3\,\overline{)\,1373} \\ -12 \\ \hline 17 \\ -15 \\ \hline 23 \\ -21 \\ \hline 2 \end{array}$$

Ex2) $\dfrac{3x^2-7x+2}{x+5}$

$$3x - 22 + \frac{112}{x+5}$$

$$\begin{array}{r} 3x - 22 \\ x+5\,\overline{)\,3x^2 - 7x + 2} \\ -(3x^2 + 15x) \\ \hline -22x + 2 \\ -(-22x - 110) \\ \hline 112 \end{array}$$

STEP#1: Determine what you can multiply the 1st term in **DIVISOR** by to get as close to the first term of the **DIVIDEND** as possible.

STEP#2: Multiply the whole **DIVISOR** by that amount

STEP #3: Subtract

STEP#4: Bring down next term

STEP#5: REPEAT

**** Note: Write your final answer as the *QUOTIENT* + $\dfrac{REMAINDER}{DIVISOR}$

Ex3) $\dfrac{2x^4-x^3-2}{2x^2+x+1}$

$$x^2 - x - \frac{-2}{2x^2+x-1}$$

$$\begin{array}{r} 2x^2+x+1\,\overline{)\,2x^4 - x^3 + 0x^2 + 0x - 2} \\ -(2x^4 + x^3 + x^2) \\ \hline -2x^3 - x^2 + 0x \\ -(-2x^3 - x^2 - x) \\ \hline x - 2 \end{array}$$

Place holders are NECESSARY when you are "missing" terms.

A Quick Summary ...The following statements are all equivalent:

1. $x = c$ is a solution (or root) of the equation $f(x) = 0$.
2. When $f(x)$ is divided by $(x - c)$, the remainder equals 0.
3. c is a zero of the function $f(x)$.
4. c is an x-intercept of the graph of $f(x)$ if c is a real number.
5. $(x - c)$ is a factor of $f(x)$.

QUESTION: So when do you get to use *SYNTHETIC DIVISION*?

➔ **ANSWER**: Whenever your **DIVISOR** is linear (aka in the form $mx+b$)

Ex4) Use BOTH long and synthetic to find the following quotient: $\dfrac{2x^3-3x^2-5x-12}{x-3}$

$x-3=0$

$x=3$

LONG DIVISION

$$
\begin{array}{r}
2x^2+3x+4 \\
x-3\,\overline{)\,2x^3-3x^2-5x-12} \\
-(2x^3-6x^2) \\
\hline
3x^2-5x \\
-(3x^2-9x) \\
\hline
4x-12 \\
-(4x-12) \\
\hline
0
\end{array}
$$

SYNTHETIC DIVISION

$$
\begin{array}{r|rrrr}
3 & 2 & -3 & -5 & -12 \\
 & & 6 & 9 & 12 \\
\hline
* & 2 & 3 & 4 & 0
\end{array}
$$

remainder

$2x^2+3x+4$

QUESTION: Why do we use long and synthetic division?

➔ **ANSWER**: Mainly to find zeros of polynomials that we could not factor using any of the methods that we have learned before.

QUESTION: If the process of long and synthetic division is "embedded" in a problem with a polynomial then how will we know what to divide by (since it will not be so specific)?

➔ **ANSWER**: Use the RATIONAL ROOT THEOREM

THEOREM: To identify all POSSIBLE RATIONAL ZEROS (again these are POSSIBLE NOT definitely zeros), we begin by listing all the factors of the constant term, factors of our leading coefficient, and then we create the list of possibilities using by finding ALL COMBINATIONS of these factors using the ones from the constant term as "numerators" and factors from leading coefficient as "denominators." Then we simplify all of these numbers, eliminate repeats, and add "plus or minus" to each of them.

Ex5) List all possible rational zeros of $f(x)=\boxed{3}x^4+2x^3-7x+6$

STEP #1: List factors of constant----------------→ 1, 2, 3, 6

STEP #2: List factors of leading coefficient-----→ 1, 3

STEP #3: Write out all combos using factors of Constant as "numerators" & factors of the Leading Coefficient as "denominators" ---------→ $1, 2, 3, 6, \frac{1}{3}, \frac{2}{3}, \cancel{1}, \cancel{2}$

STEP #4: Simplify ALL, eliminate repeats & add "plus or minus" to each---------→ $\pm 1, \pm 2, \pm 3, \pm 6, \pm\frac{1}{3}, \pm\frac{2}{3}$

possible rational roots

QUESTION: Why do we make that list again?

➔ **ANSWER**: To have some numbers to "test" to see if they are zeros of the polynomial.

Ex6) Find the zeros of $f(x) = x^3 - 3x^2 + 1$

constant: 1

l.c.: 1

possible rational roots: ± 1

$\boxed{\text{no rational roots}}$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 0 & 1 \\ & \downarrow & 1 & -2 & -2 \\ \hline & 1 & -2 & -2 & -1 \end{array}$$

rem

$x = 1$ not a root

$$\begin{array}{r|rrrr} -1 & 1 & -3 & 0 & 1 \\ & \downarrow & -1 & 4 & -4 \\ \hline & 1 & -4 & 4 & -3 \end{array}$$

rem.

$x = -1$ not a root

irrational roots

$x \approx -.532, .653, 2.879$

Ex7) Find all rational zeros of

$f(x) = 3x^3 + 4x^2 - 5x - 2$

constant: 1, 2

l.c.: 1, 3

possible rat. roots: $\pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}$

$$\begin{array}{r|rrrr} \cancel{2} & 3 & 4 & -5 & -2 \\ & \downarrow & 6 & 20 & 30 \\ \hline & 3 & 10 & 15 & 28 \end{array}$$

rem

$$\begin{array}{r|rrrr} -2 & 3 & 4 & -5 & -2 \\ & \downarrow & -6 & 4 & 2 \\ \hline & 3 & -2 & -1 & 0 \end{array}$$

rem

$x = -2$ is a root

$3x^2 - 2x - 1 = 0$

$(3x + 1)(x - 1) = 0$

$3x + 1 = 0 \quad x - 1 = 0$

$x = -\frac{1}{3} \quad x = 1$

$\boxed{\text{rational zeros: } x = 1, -\tfrac{1}{3}, -2}$

Ex8) Find all real zeros of

$f(x) = 2x^4 - 7x^3 - 8x^2 + 14x + 8$

constant: 1, 2, 4, 8

l.c.: 1, 2

possible rat. roots:

$\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}$

$$\begin{array}{r|rrrrr} 4 & 2 & -7 & -8 & 14 & 8 \\ & \downarrow & 8 & 4 & -16 & -8 \\ \hline & 2 & 1 & -4 & -2 & 0 \end{array}$$

rem

$x = 4$

$2x^3 + x^2 - 4x - 2 = 0$

$x^2(2x + 1) - 2(2x + 1) = 0$

$(2x + 1)(x^2 - 2) = 0$

$2x + 1 = 0 \quad x^2 - 2 = 0$

$x = -\frac{1}{2} \quad x^2 = 2$

$x = \pm\sqrt{2}$

$\boxed{\text{real zeros: } x = 4, -\tfrac{1}{2}, \pm\sqrt{2}}$