ICM Notes on Polynomials

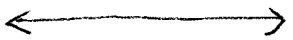
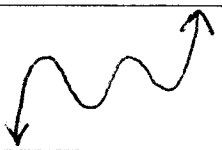
A **polynomial** has form $a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1x + a_0$

- the powers n, n-1, etc. are positive integers
- a_nx^n is the leading term
- a_n is the leading coefficient and $\neq 0$
- a_n, a_{n-1}, etc. are real numbers

examples of polynomials: $x^3 - 2x + 7$ $\quad \frac{1}{3}x^4 - \sqrt{2}$

these are not polynomials: $\frac{1}{x^2} = x^{-2}$ $\quad \sqrt{x} + 2 = x^{\frac{1}{2}} + 2$

general forms of polynomials

name	general equation	degree	graph: pos leading coefficient	graph: neg leading coefficient
constant	$y = \#$	0		
linear	$y = x$	1		
quadratic	$y = x^2$	2		
cubic	$y = x^3$	3		
quartic	$y = x^4$	4		
quintic	$y = x^5$	5		

To determine the end behavior look at limit of f(x) as $x \to -\infty$ and limit of f(x) as $x \to \infty$. In other words, determine where the ends of the graph are headed.

Example 1 Describe the end behavior.

	LEB	REB
A. $f(x) = 2x^3 + x^2 - 7$	$\lim_{x\to-\infty} f(x) = -\infty$	$\lim_{x\to\infty} f(x) = \infty$
B. $g(x) = -x^4 + 2x^3 + 3x - 3$	$\lim_{x\to-\infty} f(x) = -\infty$	$\lim_{x\to\infty} f(x) = -\infty$
C. $h(x) = 24x - 4x^5 + 2x^2 - 7$	$\lim_{x\to-\infty} f(x) = \infty$	$\lim_{x\to\infty} f(x) = -\infty$

A. cubic, l.c. positive

B. quartic, l.c. negative

C. quintic, l.c. negative

Finding zeros

repeated roots = "multiplicity"

- **if odd—the graph crosses the x-axis once**
- **if even—the graph touches the x-axis but doesn't cross through (it's tangent)**

powers on factors

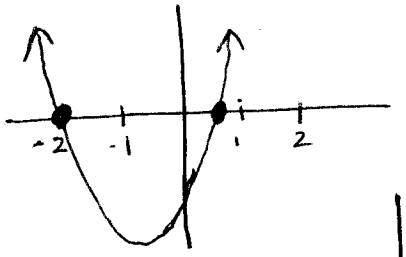
Example 2 Find the roots algebraically and sketch a graph.

A. $f(x) = 3x^2 + 4x - 4$

$(3x - 2)(x + 2) = 0$

$3x - 2 = 0 \quad x + 2 = 0$

$x = \frac{2}{3} \quad x = -2$



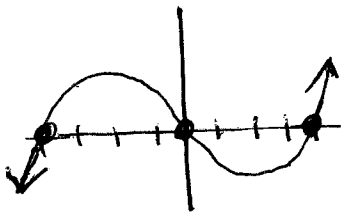
B. $g(x) = x^3 - 16x$

$x(x^2 - 16) = 0$

$x(x+4)(x-4) = 0$

$x = 0 \quad x+4 = 0 \quad x-4 = 0$

$x = 0, -4, 4$

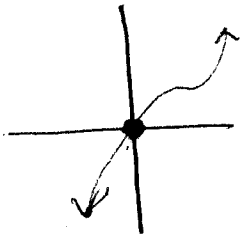


C. $f(x) = x^3 + 4x$

$x(x^2 + 4) = 0$

$x = 0 \quad x^2 + 4 = 0$

$x^2 = -4 \quad x = \pm\sqrt{-4} = \pm 2i$

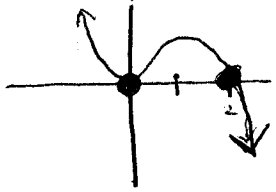


D. $y = -x^2(x - 2)$

$y = -x^3 + 2x^2$ $\quad -x^2 = 0 \quad x - 2 = 0$

$x = 0 \quad x = 2$

↳ multiplicity of 2



E. $g(x) = (x-3)^3(x-1)^2 x$

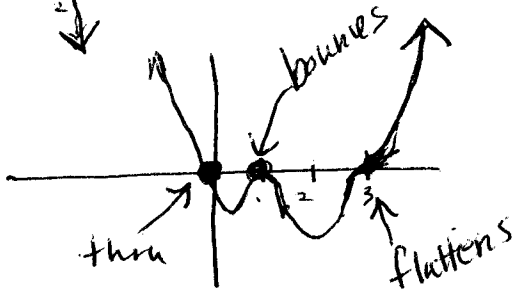
degree 6

pos l.c.

$x - 3 = 0 \quad x - 1 = 0 \quad x = 0$

$x = 3 \quad x = 1$

↳ mult. of 3 ↳ mult. of 2



Example 3 Use a calculator to find the roots: $f(x) = -x^3 + 3x^2 + 7x - 2$