This exploration is about the graphs of two particular kinds of polynomials:

Power functions: A *power function* is a polynomial of the form $f(x) = x^n$.
We've previously studied x^2, but what about other powers: x^3, x^4, x^5, etc.
What can we find out about the graph of x^n in general?

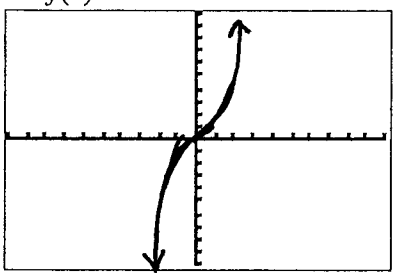
Monomials: A *monomial* is a one-term polynomial, having the form $f(x) = a \cdot x^n$.
That is, a monomial is just a power function multiplied by a number a.
For example, $f(x) = 5x^3$ and $f(x) = -2x^4$ are power functions.
What do the graphs of these functions look like?

Investigation: graphs of power functions

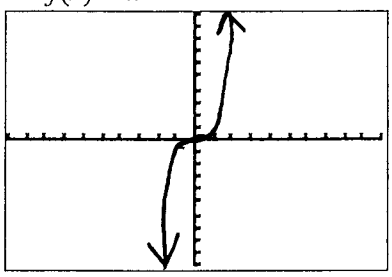
$f(x) = x^n$, n is an integer → positive ⇒ polynomi

1. a. Graph the functions shown below on your calculator, which are all of the form $f(x) = x^n$.

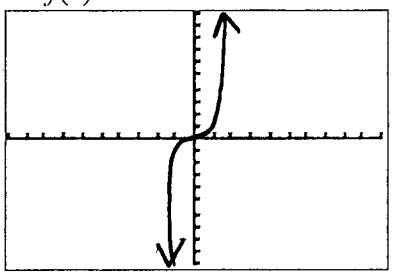
$f(x) = x^3$



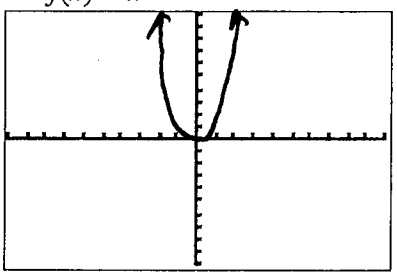
$f(x) = x^5$



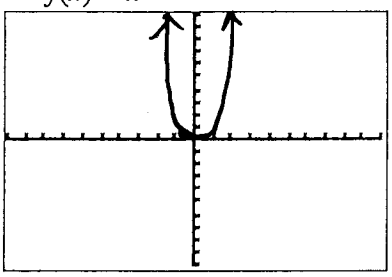
$f(x) = x^7$



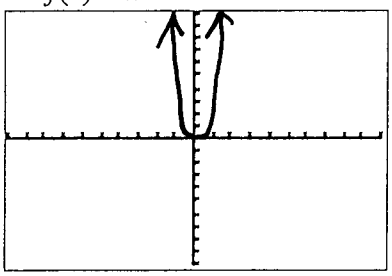
$f(x) = x^4$



$f(x) = x^6$



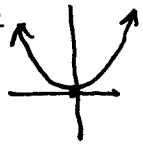
$f(x) = x^8$



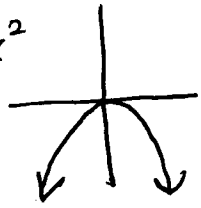
b. Explain how the n value affects the shape of the graph.

"n" is even → left & rt. ends go in the same direction

$y = x^2$

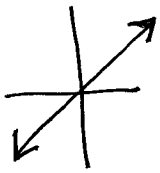


$y = -x^2$

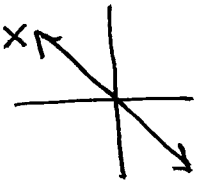


"n" is odd → left & rt. ends go in opposite directions

$y = x$



$y = -x$

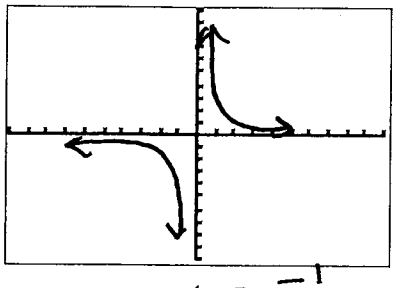


not polynomials

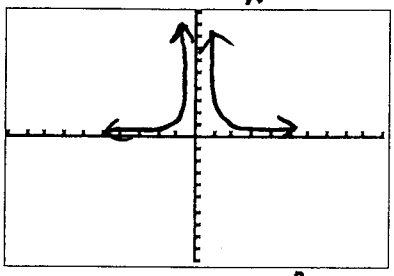
Investigations: graphs of monomials $f(x) = x^n$, n is a negative integer $\Rightarrow$ rational function

2. a. Graph the functions shown below on your calculator, which are all of the form $f(x) = ax^n$ where n is negative.

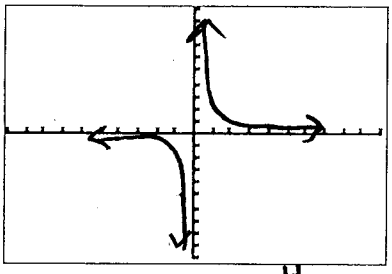
$f(x) = 2x^{-1} = \frac{2}{x}$



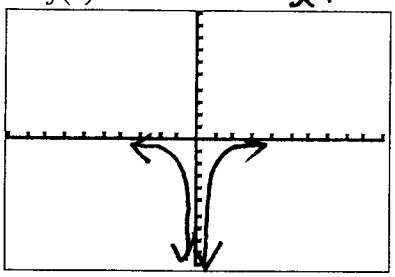
$f(x) = 4x^{-2} = \frac{4}{x^2}$



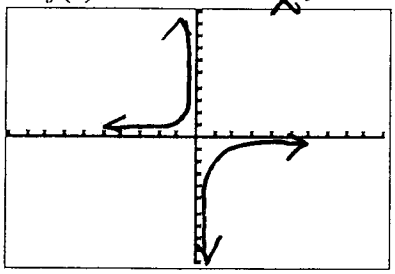
$f(x) = \frac{1}{2}x^{-3} = \frac{1}{2x^3}$



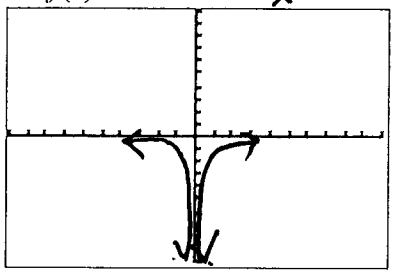
$f(x) = -1x^{-4} = \frac{-1}{x^4}$



$f(x) = -2x^{-5} = \frac{-2}{x^5}$



$f(x) = -4x^{-6} = \frac{-4}{x^6}$



b. Explain how the a value affects the shape of the graph.

n even: "a" pos 1st/2nd quad
"a" neg 3rd/4th quad
n odd: "a" pos 1st/3rd quad
"a" neg 2nd/4th quad

"a" vertical stretch or shrink

c. Explain how the n value affects the shape of the graph.

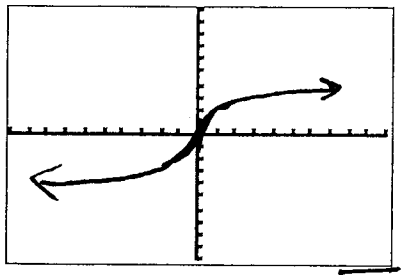
n even: symm. with respect to y-axis "even"
n odd: symm. with respect to origin "odd"

horiz. asymptote at $y = 0$
end behavior at both ends $\to 0$

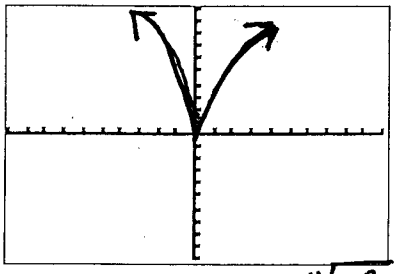
$f(x) = x^n$, n is a noninteger not polynomials

3. a. Graph the functions shown below on your calculator, which are all of the form $f(x) = ax^n$ where n is not an integer.

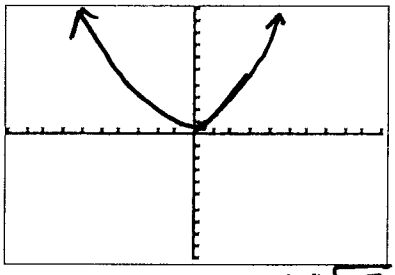
$f(x) = 2x^{1/3} = 2 \cdot \sqrt[3]{x}$



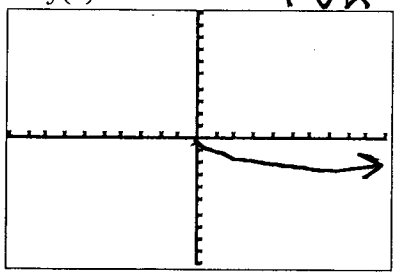
$f(x) = 4x^{2/3} = 4\sqrt[3]{x^2}$



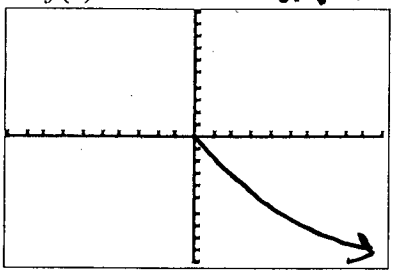
$f(x) = \frac{1}{2}x^{4/3} = \frac{1}{2}\sqrt[3]{x^4}$



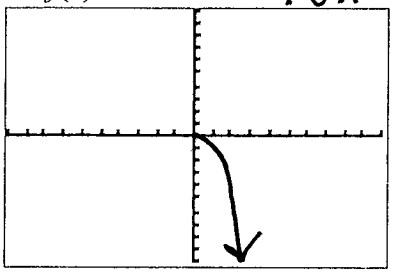
$f(x) = -1\,x^{1/4} = -1 \cdot \sqrt[4]{x}$



$f(x) = -2\,x^{3/4} = -2\sqrt[4]{x^3}$



$f(x) = -4\,x^{7/4} = -4\sqrt[4]{x^7}$



b. Explain how the a value affects the shape of the graph.

"a" → vertical stretch or shrink

odd denom ⇒ D: $(-\infty, \infty)$

even denom ⇒ D: $[0, \infty)$

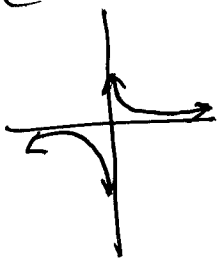
c. Explain how the n value affects the shape of the graph.

$0 < n < 1$ ends look like a root function

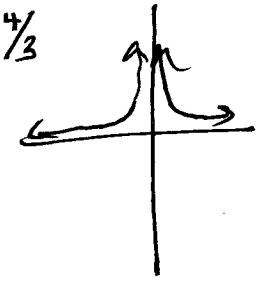
$n > 1$ ends look like a polynomial

n neg fraction → looks like a rational function

$y = 2x^{-\frac{1}{3}}$



$y = 2x^{-4/3}$



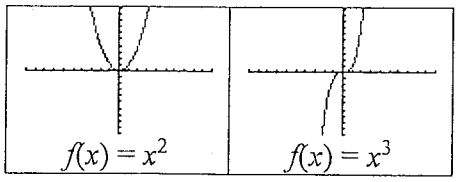
Summary: End Behavior

The term *end behavior* refers to whether each end of a graph goes up or down.
For example, the end behavior of $f(x) = x^3$ is: down on the left, up on the right.
Here is what can be seen about end behavior in the graphs you made in problems **1–3**.

Right End Behavior: The right end behavior of $f(x) = ax^n$ depends on whether *a is* positive or negative

When *a is* + , the right end behavior is ∞

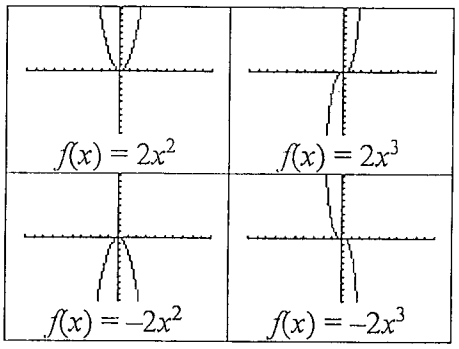
When *a is* − , the right end behavior is −∞



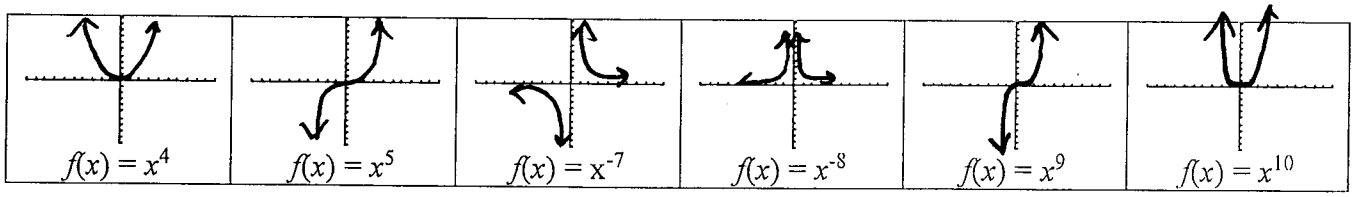
Left End Behavior: The left end behavior of $f(x) = ax^n$ depends on whether *n is* even or odd.

When *n is* even, the left end behavior is the same as the R.E.B.

When *n is* odd, the left end behavior is the opposite direction of R.E.B.



4. Without using a calculator and without looking back at earlier pages, make a rough sketch of the graph for each of the following functions of the form $f(x) = x^n$. Follow the rules about end behavior. You'll need to look at whether *n* is even or odd.



5. Again, without using a calculator, make a rough sketch of the graph for each of the following functions of the form $f(x) = x^n$. You'll need to look at whether *n* is even or odd and whether *a* is positive or negative.