Notes – Proving Trig Identities

(1) $\csc\theta \tan\theta = \sec\theta$

$$\frac{1}{\sin\theta} \cdot \frac{\sin\theta}{\cos\theta}$$

$$\frac{1}{\cos\theta}$$

$$\sec\theta$$

(2) $\tan x + \cot x = \sec x \csc x$

$$\sin x \cdot \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \cdot \cos x$$

$$\frac{\sin^2 x + \cos^2 x}{(\sin x)(\cos x)}$$

$$\frac{1}{\sin x \cos x}$$

$$\frac{1}{\sin x} \cdot \frac{1}{\cos x}$$

$$\csc x \cdot \sec x$$

(3) $\dfrac{\cos x - 2\sin x \cos x}{\cos^2 x - \sin^2 x + \sin x - 1} = \cot x$

$$\frac{\cos x\,(1 - 2\sin x)}{1 - \sin^2 x - \sin^2 x + \sin x - 1}$$

$$\frac{\cos x\,(1 - 2\sin x)}{-2\sin^2 x + \sin x}$$

$$\frac{\cos x\,(1 - 2\sin x)}{\sin x\,(-2\sin x + 1)}$$

$$\frac{\cos x}{\sin x}$$

$$\cot x$$

(4) $\cot\theta + \dfrac{1 - 2\cos^2\theta}{\sin\theta\cos\theta} = \tan\theta$

$$\cos\theta \cdot \frac{\cos\theta}{\sin\theta} + \frac{1 - 2\cos^2\theta}{\sin\theta\cos\theta}$$

$$\frac{\cos^2\theta + 1 - 2\cos^2\theta}{\sin\theta\cos\theta}$$

$$\frac{1 - \cos^2\theta}{\sin\theta\cos\theta}$$

$$\frac{\sin^2\theta}{\sin\theta\cos\theta}$$

$$\frac{\sin\theta}{\cos\theta}$$

$$\tan\theta$$

(5) $\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 2\sec x$

$\frac{\cos^2 x + 1 + \sin x + \sin x + \sin^2 x}{(1+\sin x)(\cos x)}$ (1)

$\frac{2+2\sin x}{(1+\sin x)(\cos x)}$

$\frac{2(1+\sin x)}{(1+\sin x)(\cos x)}$

$\frac{2}{\cos x}$

$2\sec x$

(6) $\frac{\tan x - \cot x}{\tan x + \cot x} = \sin^2 x - \cos^2 x$

$\frac{\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}$ (multiply by $\frac{\sin x \cos x}{\sin x \cos x}$)

$\frac{\sin^2 x - \cos^2 x}{\cos x \sin x}$

$\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}$

$\frac{\sin^2 x - \cos^2 x}{\cos x \sin x} \cdot \frac{\cos x \sin x}{\sin^2 x + \cos^2 x}$ (denominator = 1)

$\sin^2 x - \cos^2 x$

(7) $\frac{\cos x}{1-\sin x} = \frac{1+\sin x}{\cos x}$

$\left(\frac{\cos x}{1-\sin x}\right)\left(\frac{1+\sin x}{1+\sin x}\right)$

$\frac{\cos x (1+\sin x)}{1+\sin x - \sin x - \sin^2 x}$

$\frac{\cos x (1+\sin x)}{1-\sin^2 x}$

$\frac{\cos x (1+\sin x)}{\cos^2 x}$

$\frac{1+\sin x}{\cos x}$