



$P(x, f(x))$, $Q(x+h, f(x+h))$

"slope of the secant" = $\frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{x+h - x} = \frac{f(x+h) - f(x)}{h}$

"difference quotient" "average rate of change"

as Q gets closer to P, h approaches 0

"slope of the tangent line" = $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

"derivative"

"instantaneous rate of change"

Example 1 $f(x) = -3x^2$

- Find the equation of the tangent line to the curve at $x = 2$.
- Find the average rate of change for $[0, 2]$.
- Find the instantaneous rate of change at $x = 1.5$.

a) point $(2, f(2)) = (2, -12)$

$$\text{slope} = f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{-3(x+h)^2 - (-3x^2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-3x^2 - 6hx - 3h^2 + 3x^2}{h} = \lim_{h \rightarrow 0} (-6x - 3h) = -6x$$

$$f'(2) = -6(2) = -12$$

point-slope form $y - y_1 = m(x - x_1)$

$$y + 12 = -12(x - 2)$$

b) $(0, 0)$, $(2, -12)$ $\frac{-12 - 0}{2 - 0} = \boxed{-6}$

c) $f'(1.5) = -6(1.5) = \boxed{-9}$

Example 2

- Find the average rate of change for [1, 4].
- Find the slope of the secant for the points in part a.
- Find the instantaneous rate of change for $x = 2$.
- Find the equation of the tangent line at $x = 2$.
- Find the equation of the normal line at $x = 2$.

a) $(1, -3)$ $(4, 24)$ $\frac{24 - (-3)}{4 - 1} = \frac{27}{3} = \boxed{9}$

b) 9 see part a

c) $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{3(x+h)^2 - 6(x+h) - (3x^2 - 6x)}{h}$

$$= \lim_{h \rightarrow 0} \frac{3x^2 + 6hx + 3h^2 - 6x - 6h - 3x^2 + 6x}{h}$$

$$= \lim_{h \rightarrow 0} (6x + 3h - 6) = 6x - 6 \text{ general slope}$$

$$f'(2) = 6(2) - 6 = \boxed{6}$$

d) point $(2, 0)$

slope = 6 see part c

$$\boxed{y - 0 = 6(x - 2)}$$

e) Slope = $-\frac{1}{6}$
pt. $(2, 0)$

$$\boxed{y - 0 = -\frac{1}{6}(x - 2)}$$

Alternate Form of a Derivative

(can be used when seeking a value at a specific point)

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

\uparrow
 $x\text{-value}$

Example 3 Given $f(x) = x^3 - x$. Find $f'(2)$. $a = 2$

$$f'(2) = \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \rightarrow 2} \frac{x^3 - x - 6}{x - 2}$$

$$\begin{array}{r} 1 \ 0 \ -1 \ -6 \\ \downarrow 2 \ 4 \ 6 \\ 1 \ 2 \ 3 \ 0 \end{array} \quad \lim_{x \rightarrow 2} \frac{(x-2)(x^2+2x+3)}{x-2} = 4+4+3 = \boxed{11}$$

$$\text{gen. slope} = 3x^2 - 1$$

Example 4 Given $f(x) = -3x^2 + 7x + 2$. Write the equation of the tangent line at $x = 1$.

point $(1, 6)$

$$\text{slope} = f'(1) = \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1} \frac{-3x^2 + 7x + 2 - 6}{x - 1}$$

$$= \lim_{x \rightarrow 1} \frac{-3x^2 + 7x - 4}{x - 1} = \lim_{x \rightarrow 1} \frac{-1(3x^2 - 7x + 4)}{x - 1}$$

$$= \lim_{x \rightarrow 1} \frac{-1(3x - 4)(x - 1)}{x - 1} = -1(-1) = 1$$

$$y - 6 = 1(x - 1)$$

Notation

There are lots of ways to denote the derivative of a function $y = f(x)$.

$f'(x)$ the derivative of f

y' y prime

$\frac{df}{dx}$ the derivative of f with respect to x .

$\frac{dy}{dx}$ the derivative of y with respect to x .

$\frac{d}{dx} f(x)$ the derivative of f at x

