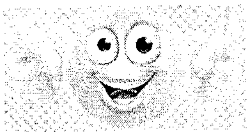

A "rational expression" is the quotient of two polynomials.

A rational expression is in *simplest form* when the numerator and denominator have no common factors other than 1.

To simplify rational expressions . . .

1) Factor the numerator and denominator completely.
2) Divide common factors.
3) State restrictions on the variable--some may no longer be visible in the final answer.

<u>Example 1</u> **Simplify.** **State any restrictions on the variable.**

A. $\dfrac{x^2+6x+5}{x^2+3x-10} = \dfrac{\cancel{(x+5)}(x+1)}{\cancel{(x+5)}(x-2)} = \dfrac{x+1}{x-2}, \quad x \neq -5, 2$

B. $\dfrac{6x^2-7x-3}{8x^2-2x-15} = \dfrac{(3x+1)\cancel{(2x-3)}}{(4x+5)\cancel{(2x-3)}} = \dfrac{3x+1}{4x+5}, \quad x \neq -\frac{5}{4}, \frac{3}{2}$

C. $\dfrac{5x}{25x^2+10x} = \dfrac{\cancel{5x}}{\cancel{5x}(5x+2)} = \dfrac{1}{5x+2}, \quad x \neq 0, -\frac{2}{5}$

D. $\dfrac{2x^2+5x-12}{15-10x} = \dfrac{\overset{-1}{\cancel{(2x-3)}}(x+4)}{5\cancel{(3-2x)}} = \dfrac{-1(x+4)}{5}, \quad x \neq \frac{3}{2}$

E. $\dfrac{y^2-36}{2y^2-7y-30} = \dfrac{(y+6)\cancel{(y-6)}}{(2y+5)\cancel{(y-6)}} = \dfrac{y+6}{2y+5}, \quad y \neq -\frac{5}{2}, 6$

F. $\dfrac{x^2-3x}{x^2-9} = \dfrac{x\cancel{(x-3)}}{\cancel{(x-3)}(x+3)} = \dfrac{x}{x+3}, \quad x \neq 3, -3$

2 1

Things to keep in mind when *multiplying or dividing* rational expressions . . .

- Only divide out FACTORS (expressions being multiplied)
- Pay close attention to location (top versus bottom)
- To divide by a fraction, multiply by the reciprocal.

Example 2 **Perform the indicated operation. Be sure to simplify and state any restrictions on the variable.**

A. $\frac{x^2-6x-7}{x^2+5x+4} \times \frac{2x^2+9x+4}{3x^2-23x+14} = \frac{\cancel{(x-7)}\cancel{(x+1)}}{\cancel{(x+4)}\cancel{(x+1)}} \bullet \frac{(2x+1)\cancel{(x+4)}}{(3x-2)\cancel{(x-7)}} = \frac{2x+1}{3x-2}$

$x \neq -1, -4, \frac{2}{3}, 7$

B. $\frac{2x^2+11x-21}{x^3+2x^2+4x} \cdot \frac{x^3-8}{x^2+5x-14} = \frac{(2x-3)\cancel{(x+7)}}{x\cancel{(x^2+2x+4)}} \bullet \frac{\cancel{(x-2)}\cancel{(x^2+2x+4)}}{\cancel{(x+7)}\cancel{(x-2)}} = \frac{2x-3}{x}$

$x \neq 0, -7, 2$

C. $4(c^2+2cd+d^2)$

$\frac{4c^2+8cd+4d^2}{8} \bullet \frac{3c-3d}{c^2-d^2} = \frac{\overset{1}{\cancel{4}}(c+d)\cancel{(c+d)}}{\cancel{8}_2} \bullet \frac{3\cancel{(c-d)}}{\cancel{(c-d)}\cancel{(c+d)}} = \frac{3(c+d)}{2}$

$c-d \neq 0$ $c+d \neq 0$

$c \neq d$ $c \neq -d$

D. $\frac{x^2-9}{x^2+5x+4} \div \frac{x^2+4x+3}{x^2+7x+12} = \frac{(x+3)(x-3)}{\cancel{(x+4)}(x+1)} \bullet \frac{\cancel{(x+4)}\cancel{(x+3)}}{\cancel{(x+3)}(x+1)} = \frac{(x+3)(x-3)}{(x+1)^2}$

$x \neq -4, -1, -3$

E. $\frac{3t^2+10t-8}{4t^2-12t+9} \div \frac{6t^2-13t+6}{4t^2-9} = \frac{\cancel{(3t-2)}(t+4)}{(2t-3)(2t-3)} \bullet \frac{(2t+3)\cancel{(2t-3)}}{\cancel{(3t-2)}(2t-3)} = \frac{(t+4)(2t+3)}{(2t-3)^2}$

$t \neq \frac{3}{2}, \frac{2}{3}, -\frac{3}{2}$

F. $\frac{x^3+1}{x^2-x-2} \div \frac{x^2-x+1}{x^2-4x+4} = \frac{\cancel{(x+1)}\cancel{(x^2-x+1)}}{\cancel{(x-2)}\cancel{(x+1)}} \bullet \frac{\cancel{(x-2)}(x-2)}{\cancel{(x^2-x+1)}} = x-2$

$x \neq 2, -1$