DEFINITION: A **Series** is the sum of the terms of a sequence.

Sigma Notation or (Summation Notation) This symbol means "Add" → $\sum$ It's called "Sigma"

$$\sum_{n=1}^{k} a_n$$

The variable below sigma (n in this case) is called the **"index"**
The number below sigma (1 in this case) is which term begins the series, called the **"lower bound"**
The number above sigma (k in this case) is which term ends the series, called the **"upper bound"**
The expression to the right of sigma (a_n in this case) is the **explicit formula** used to generate the terms of the series.

Ex 1) Find the sum of each finite sequence.

(a) $\sum_{n=1}^{5} n^2 = 1+4+9+16+25 = 55$

(b) $\sum_{n=3}^{5} \frac{1}{n} = \frac{1}{3}+\frac{1}{4}+\frac{1}{5} = \frac{47}{60}$

(c) $\sum_{n=5}^{10} n = 5+6+7+8+9+10 = 45$

(d) $\sum_{n=1}^{6} 2 = 2+2+2+2+2+2 = 12$

Ex2) Write the following sums using sigma notation:

a) $1+4+9+16+25+36+49+64+81$

$\sum_{n=1}^{9} n^2$

b) $2+4+6+8+10+12$

$\sum_{n=1}^{6} 2n$

arith.
$d=2$
$a_1=2$
$a_n=2+2(n-1)$
$a_n=2n$

c) $625+125+25+\cdots$

$\sum_{n=1}^{\infty} 625\left(\frac{1}{5}\right)^{n-1}$

geom
$r=\frac{1}{5}$
$a_1=6$
$a_n=6$

d) $6+2-2-6-10-14-18-22$

$\sum_{n=1}^{8} (-4n+10)$

arith.
$d=-4$
$a_1=6$
$a_n=6+-4(n-1)$

e) $729+243+81+27+9+3$

$\sum_{n=1}^{6} 729\left(\frac{1}{3}\right)^{n-1}$

f) $8+27+64+125$

$\sum_{n=2}^{5} n^3$

FINITE ARITHMETIC SERIES $S_n = \sum_{k=1}^{n} a_k = \frac{n}{2}(a_1 + a_n)$

$n=$ # terms
$a_1=$ 1st term
$a_n=$ last term

Ex3) Find each sum.

a) $3+7+11+15+19+23+27$

$\frac{7}{2}(3+27) = \boxed{105}$

b) $1+3+5+7+\cdots+49$

$a_n = a_1 + d(n-1)$
$49 = 1+2(n-1)$
$48 = 2(n-1)$
$24 = n-1 \quad n=25$

$\frac{25}{2}(1+49) = \boxed{625}$

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Ex4) A corner section of a stadium has 8 seats along the front row. Each successive row has 2 more seats than the row preceding it. If the top row has 24 seats how many seats are in the entire section?

$8+10+12+\ldots+24$

$24=8+2(n-1)$

$16=2(n-1)$

$8=n-1$

$9=n$

$\frac{9}{2}(8+24)=\boxed{144 \text{ seats}}$

Sum of a Finite Geometric Series:

$$S_n=a_1\frac{1-r^n}{1-r}$$

Sum of an Infinite Geometric Series:

$$\sum_{k=1}^{\infty}a_1\cdot r^{n-1}=\frac{a_1}{1-r}$$, if & only if $|r|<1$

If an infinite series has a sum it is convergent.

If it does not, it is divergent.

Ex5) Find the sum of each geometric series.

a) $\sum_{n=1}^{9}4\left(-\frac{1}{3}\right)^{n-1}=4+-\frac{4}{3}+\frac{4}{9}+\ldots+\frac{4}{6561}$

$4\cdot\frac{1-\left(-\frac{1}{3}\right)^9}{1-\left(-\frac{1}{3}\right)}=\boxed{\frac{19684}{6561}}$

b) $5+15+45+\ldots+98415$

$98415=5(3)^{n-1}$

$19683=3^{n-1}$

$\ln 19683=(n-1)\ln 3$

$9=n-1$

$10=n$

$5\cdot\frac{1-(3)^{10}}{1-3}$

$\boxed{147620}$

Ex6) Determine if each of the following series converges. If the series converges, find the sum.

a) $\sum_{n=1}^{\infty}3(0.75)^{n-1}$

$r=0.75 \quad |0.75|=.75<1$ ✓

Series converges

$\frac{3}{1-0.75}=\boxed{12}$

b) $\sum_{n=0}^{\infty}\left(-\frac{4}{5}\right)^{n-1}=-\frac{5}{4}+1+\frac{-4}{5}+\frac{16}{25}+\ldots$

$r=-\frac{4}{5} \quad \left|-\frac{4}{5}\right|=\frac{4}{5}<1$ ✓

$\frac{-\frac{5}{4}}{1-\left(-\frac{4}{5}\right)}=\boxed{\frac{-25}{36}}$

c) $\sum_{n=1}^{\infty}\left(\frac{\pi}{2}\right)^n=\frac{\pi}{2}+\frac{\pi^2}{4}+\frac{\pi^3}{8}+\frac{\pi^4}{16}+\ldots$

$r=\frac{\pi}{2} \quad \left|\frac{\pi}{2}\right|>1$

series diverges

$\boxed{\text{no sum}}$

d) $\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1}=1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$

$r=\frac{1}{2} \quad \left|\frac{1}{2}\right|=\frac{1}{2}<1$ ✓

Series Converges

$\frac{1}{1-\frac{1}{2}}=\boxed{2}$