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# Notes (9.4) Part 3 – Sigma Notation and Arithmetic Series

**DEFINITION:** A **Series** is the sum of the terms of a sequence.

**Sigma Notation or (Summation Notation)** This symbol means "Add" → $\sum$ It's called "Sigma"

$$\sum_{n=1}^{k} a_n$$

The variable below sigma ($n$ in this case) is called the **"index"**
The number below sigma (1 in this case) is which term begins the series, called the **"lower bound"**
The number above sigma (k in this case) is which term ends the series, called the **"upper bound"**
The expression to the right of sigma ($a_n$ in this case) is the **explicit formula** used to generate the terms of the series.

**Ex 1) Using summation notation to find the sum of a finite sequence**

| Problem | Work | Answer |
|---|---|---|
| (a) $\sum_{n=1}^{5} n^2$ | $1^2+2^2+3^2+4^2+5^2$<br>$1+4+9+16+25$ | $\boxed{55}$ |
| (b) $\sum_{n=3}^{5} \frac{1}{n}$ | $\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$ | $\boxed{\frac{47}{60}}$ |
| (c) $\sum_{n=5}^{10} n$ | $5+6+7+8+9+10$ | $\boxed{45}$ |
| (d) $\sum_{n=1}^{6} 2$ | $2+2+2+2+2+2$ | $\boxed{12}$ |

**Ex2) Write the following sums using sigma notation:**

**a)** $1+4+9+16+25+36+49+64+81$

$\sum_{n=1}^{9} n^2$

**b)** $2+4+6+8+10+12$ $\sum_{n=1}^{6} 2n$

arith. $a_n = a_1 + d(n-1)$
$d = 2$ $2+2(n-1)$
$a_1 = 2$ $2+2n-2$
$2n$

**c)** $625+125+25+\cdots$ $\sum_{n=0}^{\infty} \frac{625}{5^n}$

geom. $r=\frac{1}{5}$
$a_n = a_1(r)^{n-1}$
$\sum_{n=1}^{\infty} 625\left(\frac{1}{5}\right)^{n-1}$

**Now You Try ☺**

**a)** $6+2-2-6-10-14-18-22$ $\sum_{n=1}^{8}(-4n+10)$

arith. $a_1=6$ $a_n = 6+-4(n-1)$
$d=-4$ $a_n = -4n+10$

**b)** $729+243+81+27+9+3$

geom. $r=\frac{1}{3}$ $\sum_{n=1}^{6} 729\left(\frac{1}{3}\right)^{n-1}$

**c)** $8+27+64+125$ $\sum_{n=2}^{5} n^3$

**FINITE ARITHMETIC SERIES** $S_n = \sum_{k=1}^{n} a_k = \frac{n}{2}(a_1 + a_n)$ Gauss→$1+2+3+4+\cdots+100$

**a)** $1+3+5+7+\cdots+49$

$a_n = a_1 + d(n-1)$
$49 = 1+2(n-1)$
$n=25$

$S_{25} = \frac{25}{2}(1+49)$ $\boxed{625}$

**b)** $3+7+11+15+19+23+27$

$S_7 = \frac{7}{2}(3+27) = \boxed{105}$

**c)** A corner section of a stadium has 8 seats along the front row. Each successive row has 2 more seats than the row preceding it. If the top row has 24 seats, how many seats are in the entire section?

$8+10+12+\ldots+24$

$24 = 8+2(n-1)$
$n=9$

$S_9 = \frac{9}{2}(8+24) = \boxed{144 \text{ seats}}$

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# Notes (9.4 Part 4) – Geometric Series

## Sum of a Finite Geometric Series:

$$S_n = a_1 \frac{1-r^n}{1-r}$$

## Sum of an Infinite Geometric Series:

$$\sum_{k=1}^{\infty} a_1 \cdot r^{n-1} = \frac{a_1}{1-r}$$, if & only if |r|<1

If an infinite series has a sum, it is convergent

If it does not, it is called divergent

**Ex1)** Find the sum of the given geometric series

**a)** $\sum_{n=1}^{11} 4\left(-\frac{1}{3}\right)^{n-1}$

$$S_{11} = 4 \cdot \frac{1-(-\frac{1}{3})^{11}}{1-(-\frac{1}{3})}$$

$$= \boxed{3.000016935}$$

**b)** 5 + 15 + 45 + …+ 98415

$a_n = a_1 (r)^{n-1}$

$98415 = 5(3)^{n-1}$

$19683 = 3^{n-1}$

$19683 = \frac{3^n}{3^1}$

$\log(59049) = \log(3^n)$ $n = 10$

$$S_{10} = 5 \cdot \frac{1-3^{10}}{1-3}$$

$$= \boxed{147620}$$

**Ex2)** Determine if the following series converge. If they do converge, find the sum:

**a)** $\sum_{n=1}^{\infty} 3(0.75)^{n-1}$

$r = .75$ $|.75| = .75 < 1$

series converges

$$S = \frac{3}{1-.75} = \boxed{12}$$

**b)** $\sum_{n=0}^{\infty}\left(-\frac{4}{5}\right)^{n-1} = -\frac{5}{4} + 1 + \frac{-4}{5} + \frac{16}{25} + \ldots$

$r = -\frac{4}{5}$ $a_1 = -\frac{5}{4}$ $\left|-\frac{4}{5}\right| = \frac{4}{5} < 1$

converges

$$S = \frac{-\frac{5}{4}}{1-(-\frac{4}{5})} = \boxed{-\frac{25}{36}}$$

**c)** $\sum_{n=1}^{\infty}\left(\frac{\pi}{2}\right)^{n} = \frac{\pi}{2} + \frac{\pi^2}{4} + \frac{\pi^3}{8} + \frac{\pi^4}{16} + \ldots$

$r = \frac{\pi}{2}$ $\left|\frac{\pi}{2}\right| = \frac{\pi}{2} > 1$

series diverges

no sum possible

**d)** $\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

$r = \frac{1}{2}$ $\left|\frac{1}{2}\right| = \frac{1}{2} < 1$

converges

$$S = \frac{1}{1-\frac{1}{2}} = \boxed{2}$$