NOTES--Graphs of Sine and Cosine

VOCABULARY

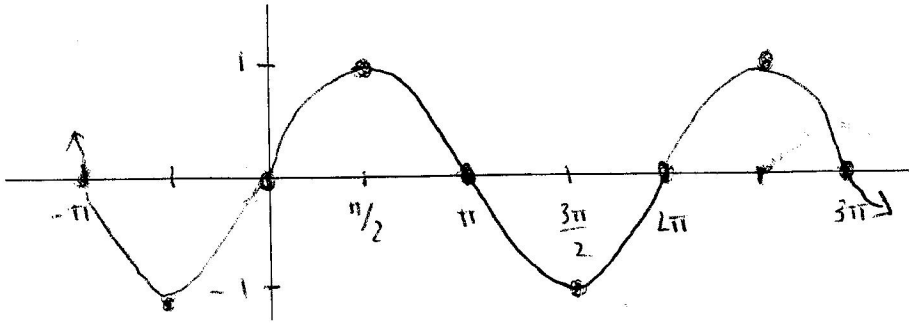
Periodic A function whose graph has a repeating pattern that continues indefinitely

Cycle The shortest repeating portion of a periodic function

Period The horizontal length of each cycle of a periodic function

Amplitude The amplitude of the graph of a sine or cosine function is $\frac{1}{2}(M - m)$ where M is the maximum value of the function and m is the minimum value of the function.

$y = \sin\theta$

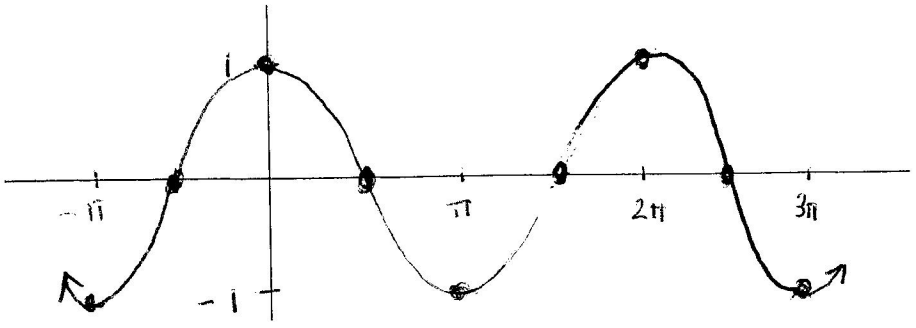


θ	sinθ
0	0
π/2	1
π	0
3π/2	-1
2π	0

domain: $(-\infty, \infty)$ amplitude: 1

range: $[-1, 1]$ period: 2π

$y = \cos\theta$



θ	cosθ
0	1
π/2	0
π	-1
3π/2	0
2π	1

domain: $(-\infty, \infty)$ amplitude: 1

range: $[-1, 1]$ period: 2π

Graphs of Sinusoids

The graphs of $y = a \sin(b(x - h)) + k$ and $y = a \cos(b(x - h)) + k$ (where $a \neq 0$ and $b \neq 0$) have the following characteristics:

amplitude $= |a|$;

period $= \frac{2\pi}{|b|}$;

frequency $= \frac{|b|}{2\pi}$.

When compared to the graphs of $y = a \sin bx$ and $y = a \cos bx$, respectively, they also have the following characteristics:

a phase shift of h;

a vertical translation of k.

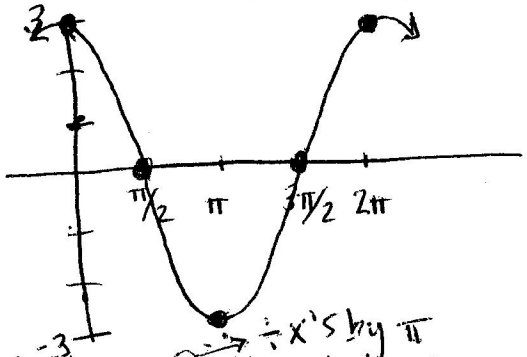
For the graphs of $y = A\sin(Bx - C) + D$ and $y = A\cos(Bx - C) + .$

- Amplitude $= |A|$
- Period $= \frac{2\pi}{B}$
- Phase Shift $= \frac{C}{B}$
- Vertical Shift $= D$
- Distance Between Key Points* $= \left(\frac{1}{4}\right) \cdot (\text{period})$

Key Points are the points that are at the top or bottom of the graph, or the points on the center-line of the graph

Example 1 Graph y = 3cosx. Name the domain, range, amplitude, and period. Describe the transformations applied to the parent graph.

x	y
0	3
π/2	0
π	-3
3π/2	0
2π	3



D: $(-\infty, \infty)$

R: $[-3, 3]$

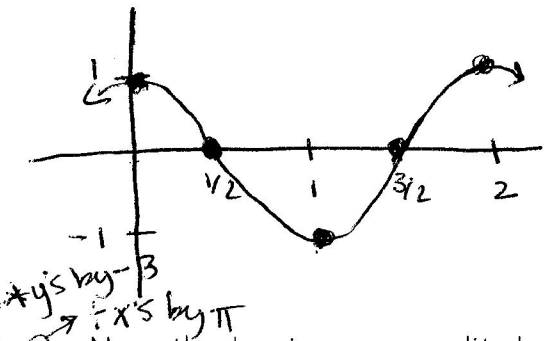
amp $= \frac{1}{2}(6) = 3$

per $= 2\pi$

vertical stretch *

Example 2 Graph y = cosπx. Name the domain, range, amplitude, and period. Describe the transformations applied to the parent graph. horiz. shrink * $\frac{1}{\pi}$

0	1
1/2	0
1	-1
3/2	0
2	1



D: $(-\infty, \infty)$

R: $[-1, 1]$

amp $= 1$

per $= \frac{2\pi}{|\pi|} = 2$

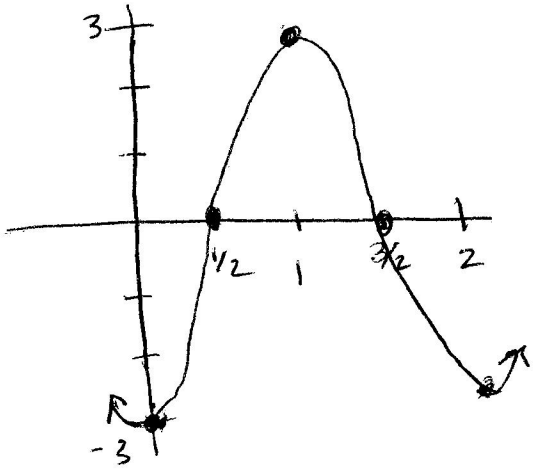
Example 3 Graph y = -3cosπx. Name the domain, range, amplitude, and period. Describe the transformations applied to the parent graph.

horiz. shrink * $\frac{1}{\pi}$

vertical stretch * 3

reflection over x-axis

0	-3
1/2	0
1	3
3/2	0
2	-3



D: $(-\infty, \infty)$

R: $[-3, 3]$

amp $= |-3| = 3$

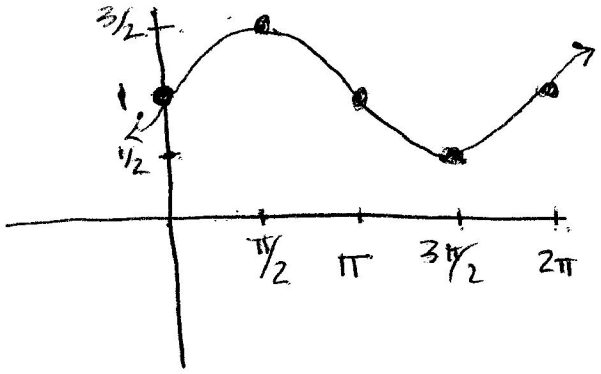
per $= \frac{2\pi}{|\pi|} = 2$

Example 4 Graph $y = \frac{1}{2}\sin(x) + 1$. Name the domain, range, amplitude, and period. Describe the transformations applied to the parent graph.

* y's by $\frac{1}{2}$ — add 1 to y's

Vertical shrink * $\frac{1}{2}$
Shift up 1

0	$0 \cdot \frac{1}{2} + 1 = 1$
$\pi/2$	$1 \cdot \frac{1}{2} + 1 = \frac{3}{2}$
π	$0 \cdot \frac{1}{2} + 1 = 1$
$3\pi/2$	$-1 \cdot \frac{1}{2} + 1 = \frac{1}{2}$
2π	$0 \cdot \frac{1}{2} + 1 = 1$



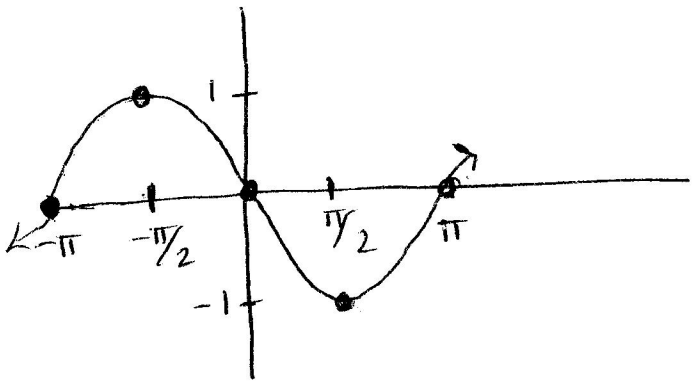
D: $(-\infty, \infty)$
R: $[\frac{1}{2}, \frac{3}{2}]$
amp $= |\frac{1}{2}| = \frac{1}{2}$
per $= \frac{2\pi}{|1|} = 2\pi$

Example 5 Graph $y = \sin(x + \pi)$. Name the domain, range, amplitude, and period. Describe the transformations applied to the parent graph.

subtract π from x's

Shift left π

x	y
$-\pi$	0
$-\frac{1}{2}\pi$	1
0	0
$\frac{1}{2}\pi$	−1
π	0



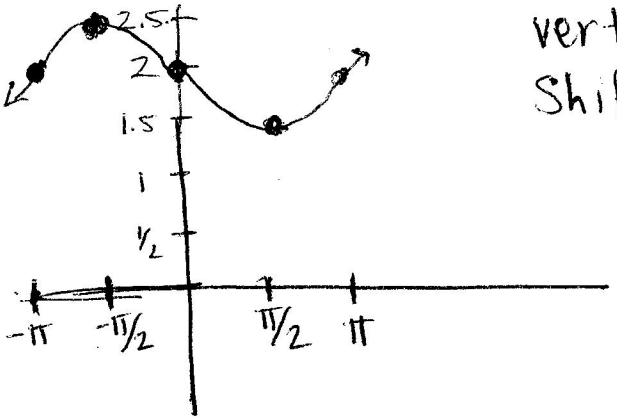
D: $(-\infty, \infty)$
R: $[-1, 1]$
amp $= |1| = 1$
per $= 2\pi$

Example 6 Graph $y = \frac{1}{2}\sin(x + \pi) + 2$. Name the domain, range, amplitude, and period. Describe the transformations applied to the parent graph.

mult. y's by $\frac{1}{2}$ — subtract π from x's — add 2 to y's

Shift left π
vert. shrink * $\frac{1}{2}$
Shift up 2

$-\pi$	2
$-\frac{\pi}{2}$	$2\frac{1}{2}$
0	2
$\frac{\pi}{2}$	$1\frac{1}{2}$
π	2



D: $(-\infty, \infty)$
R: $[1.5, 2.5]$
amp $= |\frac{1}{2}| = \frac{1}{2}$
per $= 2\pi$