Calculus AB Multiple Choice Practice Solutions **Name:** ____________________

Multiple Choice - Non-Calculator

1. $\lim\limits_{x\to\infty} \dfrac{(2x-1)(3-x)}{(x-1)(x+3)}$ is EBM: $\dfrac{-2x^2}{x^2} = -2$

B

(A) -3 (B) -2 (C) 2 (D) 3 (E) nonexistent

2. $\int \frac{1}{x^2}\,dx =$ $\int x^{-2}\,dx = -x^{-1} + C = \dfrac{-1}{x} + C$

D

(A) $\ln x^2 + C$ (B) $-\ln x^2 + C$ (C) $x^{-1} + C$ (D) $-x^{-1} + C$ (E) $-2x^{-3} + C$

3. If $f(x) = (x-1)(x^2+2)^3$, then $f'(x) =$

(A) $6x(x^2+2)^2$ $(x-1)3(x^2+2)^2 \cdot 2x + (x^2+2)^3(1)$

(B) $6x(x-1)(x^2+2)^2$ $(x^2+2)^2(6x(x-1)+x^2+2)$

D

(C) $(x^2+2)^2(x^2+3x-1)$ $(x^2+2)^2(6x^2-6x+x^2+2)$

(D) $(x^2+2)^2(7x^2-6x+2)$ $(x^2+2)^2(7x^2-6x+2)$

(E) $-3(x-1)(x^2+2)^2$

4. $\int(\sin(2x)+\cos(2x))\,dx =$

(A) $\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+C$ $-\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+C$

B

(B) $-\frac{1}{2}\cos(2x)+\frac{1}{2}\sin(2x)+C$

(C) $2\cos(2x)+2\sin(2x)+C$

(D) $2\cos(2x)-2\sin(2x)+C$

(E) $-2\cos(2x)+2\sin(2x)+C$

5. $\lim\limits_{x\to 0} \dfrac{5x^4+8x^2}{3x^4-16x^2}$ is $= \dfrac{0}{0}$ $\lim\limits_{x\to 0} \dfrac{x^2(5x^2+8)}{x^2(3x^2-16)} = \dfrac{8}{-16} = \dfrac{-1}{2}$

(A) $-\frac{1}{2}$ A (B) 0 (C) 1 (D) $\frac{5}{3}+1$ (E) nonexistent

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$$

6. Let f be the function defined above. Which of the following statements about f are true?

✓ I. f has a limit at $x = 2$. $\lim\limits_{x\to 2} \dfrac{x^2-4}{x-2} = \dfrac{0}{0}$ $\lim\limits_{x\to 2} \dfrac{2x}{1} = 4$

X II. f is continuous at $x = 2$.

X III. f is differentiable at $x = 2$. $4 \neq 1$ so not continuous

(A) I only A not continuous so not diff

(B) II only

(C) III only

(D) I and II only

(E) I, II, and III

7. A particle moves along the x-axis with velocity given by $v(t) = 3t^2 + 6t$ for time $t \geq 0$. If the particle is at position $x = 2$ at time $t = 0$, what is the position of the particle at $t = 1$?

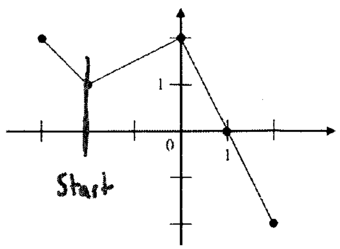
(A) 4 (B) 6 B (C) 9 (D) 11 (E) 12

$2 + \int_0^1 v(t)\,dt = 2 + t^3 + 3t^2 \Big|_0^1 = 2 + 1 + 3 = 6$

8. If $f(x) = \cos(3x)$, then $f'\left(\frac{\pi}{9}\right) =$ $f'(x) = -\sin(3x) \cdot 3$

(A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{3}{2}$ E (E) $-\frac{3\sqrt{3}}{2}$

$f'\left(\frac{\pi}{9}\right) = -\sin\left(\frac{\pi}{3}\right) \cdot 3 = -\frac{\sqrt{3}}{2} \cdot 3 = -\frac{3\sqrt{3}}{2}$

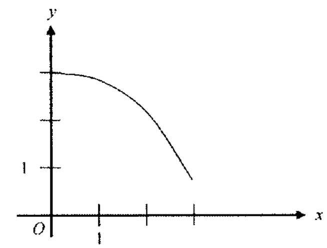


Graph of f

9. The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-2}^{x} f(t)\,dt$, which of the following values is greatest?

$-$ 0 + D larger + smaller + than $g(1)$

(A) $g(-3)$ (B) $g(-2)$ (C) $g(0)$ (D) $g(1)$ (E) $g(2)$



Graph of f

10. The graph of function f is shown above for $0 \leq x \leq 3$. Of the following, which has the least value?

(A) $\int_1^3 f(x)\,dx$

decreasing so left > right
right is an underestimate

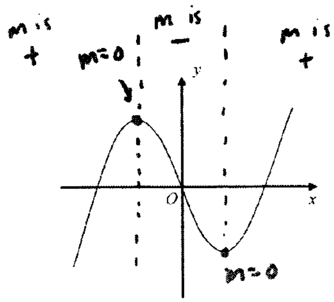
(B) Left Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

C

(C) Right Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(D) Midpoint Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

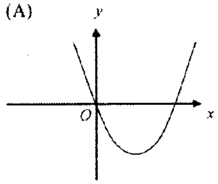
(E) Trapezoidal sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length



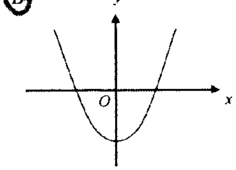
Graph of f

11. The graph of a function f is shown above. Which of the following could be the graph of f', the derivative of f?

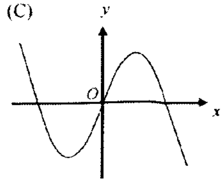
(A)



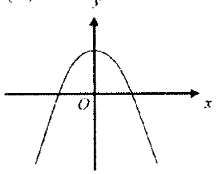
(B) B



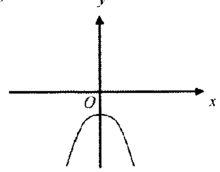
(C)



(D)



(E)



12. If $f(x) = e^{(2/x)}$, then $f'(x) =$

(A) $2e^{(2/x)}\ln x$ (B) $e^{(2/x)}$ (C) $e^{(-2/x^2)}$ (D) $-\frac{2}{x^2}e^{(2/x)}$ (E) $-2x^2e^{(2/x)}$

D

$$\begin{array}{c|c} e^x & e^x \\ \frac{2}{x} & \frac{-2}{x^2} \end{array}$$

$$e^{\frac{2}{x}} \cdot \frac{-2}{x^2} = \frac{-2e^{\frac{2}{x}}}{x^2}$$

13. If $f(x) = x^2 + 2x$, then $\frac{d}{dx}(f(\ln x)) =$ $f(\ln x) = (\ln x)^2 + 2(\ln x)$

(A) $\frac{2\ln x + 2}{x}$ (B) $2x\ln x + 2$ (C) $2\ln x + 2$ (D) $2\ln x + \frac{2}{x}$ (E) $\frac{2x+2}{x}$

A

$$\frac{d}{dx}(f(\ln x)) = 2(\ln x)\cdot\frac{1}{x} + 2\cdot\frac{1}{x} = \frac{2\ln x}{x} + \frac{2}{x}$$

x	0	1	2	3
$f''(x)$	5	0	−7	4

14. The polynomial function f has selected values of its second derivative f'' given in the table above. Which of the following statements must be true?

(A) f is increasing on the interval $(0, 2)$.

(B) f is decreasing on the interval $(0, 2)$.

(C) f has a local maximum at $x = 1$.

(D) The graph of f has a point of inflection at $x = 1$.

(E) The graph of f changes concavity in the interval $(0, 2)$. E

Even though $f''(1)$ is 0, you do not know whether f'' changes sign at 1 because you do not know what $f(x)$ does immediately to the left or right of $x=1$.

15. $\int \frac{x}{x^2-4}\,dx =$

(A) $\frac{-1}{4(x^2-4)^2}+C$

(B) $\frac{1}{2(x^2-4)}+C$

(C) $\frac{1}{2}\ln|x^2-4|+C$ C

(D) $2\ln|x^2-4|+C$

(E) $\frac{1}{2}\arctan\left(\frac{x}{2}\right)+C$

$u = x^2-4 \quad du = 2x\,dx \quad dx = \frac{du}{2x}$

$\int \frac{x}{u}\cdot\frac{du}{2x} = \frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln|u|+c$

$\frac{1}{2}\ln|x^2-4|+c$

16. If $\sin(xy) = x$, then $\frac{dy}{dx} =$

(A) $\frac{1}{\cos(xy)}$

(B) $\frac{1}{x\cos(xy)}$

(C) $\frac{1-\cos(xy)}{\cos(xy)}$

D

(D) $\frac{1-y\cos(xy)}{x\cos(xy)}$

(E) $\frac{y(1-\cos(xy))}{x}$

$\cos(xy)(xy'+y) = 1$

$xy'+y = \frac{1}{\cos(xy)}$

$xy' = \frac{1}{\cos(xy)} - y$

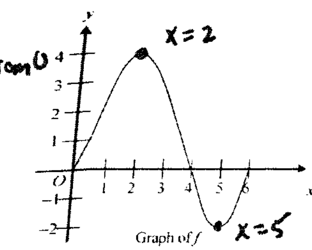
$y' = \frac{1}{x\cos(xy)} - \frac{y}{x}$

$y' = \frac{1-y\cos(xy)}{x\cos(xy)}$

g F Area from 0

g' f y-vals

g'' f' slope



Look for where f changes slope sign

17. The graph of the function f shown above has horizontal tangents at $x = 2$ and $x = 5$. Let g be the function defined by $g(x) = \int_0^x f(t)\,dt$. For what values of x does the graph of g have a point of inflection?

C

(A) 2 only (B) 4 only (C) 2 and 5 only (D) 2, 4, and 5 (E) 0, 4, and 6

18. In the xy-plane, the line $x + y = k$, where k is a constant, is tangent to the graph of $y = x^2 + 3x + 1$. What is the value of k?

(A) -3 A (B) -2 (C) -1 (D) 0 (E) 1

$x+y=k$

$y=-x+k$

$m=-1$

$y=x^2+3x+1$

$y'=2x+3$

$-1=2x+3$

$2x=-4$

$x=-2$

$y(-2)=-1$

$x+y=k$

$-2+-1=-3$

19. What are all horizontal asymptotes of the graph of $y = \frac{5+2^x}{1-2^x}$ in the xy-plane?

(A) $y=-1$ only

(B) $y=0$ only

(C) $y=5$ only E

(D) $y=-1$ and $y=0$

(E) $y=-1$ and $y=5$

$\lim_{x\to\infty} \frac{5+2^x}{1-2^x}$ EBM. $\frac{2^x}{-2^x} = -1$

$\lim_{x\to-\infty} \frac{5+2^x}{1-2^x}$ EBM: $\frac{5}{1} = 5$

20. Let f be a function with a second derivative given by $f''(x) = x^2(x-3)(x-6)$. What are the x-coordinates of the points of inflection of the graph of f?

(A) 0 only

(B) 3 only

(C) 0 and 6 only

(D) 3 and 6 only

(E) 0, 3, and 6

f'': $+$ $+$ $-$ $+$

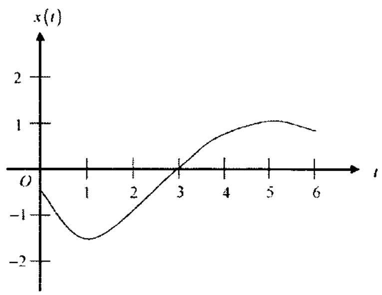
f: $\cup$ 0 $\cup$ 3 $\cap$ 6 $\cup$

D

pos. y-vals

v(t) slope

a(t) concavity



21. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

(A) $0 < t < 2$

(B) $1 < t < 5$

(C) $2 < t < 6$

(D) $3 < t < 5$ only

(E) $1 < t < 2$ and $5 < t < 6$

A

when is acceleration > 0?

concave up

22. A rumor spreads among a population of N people at a rate proportional to the product of the number of people who have heard the rumor and the number of people who have not heard the rumor. If p denotes the number of people who have heard the rumor, which of the following differential equations could be used to model this situation with respect to time t, where k is a positive constant?

(A) $\frac{dp}{dt} = kp$

(B) $\frac{dp}{dt} = kp(N-p)$

(C) $\frac{dp}{dt} = kp(p-N)$

(D) $\frac{dp}{dt} = kt(N-t)$

(E) $\frac{dp}{dt} = kt(t-N)$

B

$$\frac{dP}{dt} = kP(N-P)$$

23. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{x^2}{y}$ with the initial condition $y(3) = -2$?

(A) $y = 2e^{-9+x^3/3}$

(B) $y = -2e^{-9+x^3/3}$

(C) $y = \sqrt{\frac{2x^3}{3}}$

(D) $y = \sqrt{\frac{2x^3}{3} - 14}$

(E) $y = -\sqrt{\frac{2x^3}{3} - 14}$

E

$y\,dy = x^2\,dx$

$\frac{1}{2}y^2 = \frac{1}{3}x^3 + C$

$y^2 = \frac{2}{3}x^3 + C$

$y = \pm\sqrt{\frac{2}{3}x^3 + C}$

$y = -\sqrt{\frac{2}{3}x^3 - 14}$

$-2 = -\sqrt{\frac{2}{3}(3)^3 + C}$

$-2 = -\sqrt{18 + C}$

$2 = \sqrt{18 + C}$

$4 = 18 + C$

$C = -14$

24. The function f is twice differentiable with $f(2)=1$, $f'(2)=4$, and $f''(2)=3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x=2$?

(A) 0.4 (B) 0.6 B (C) 0.7 (D) 1.3 (E) 1.4

$y-1=4(x-2)$ $y-1=4(1.9-2)$ $y-1=4(-.1)$ $y=.6$

$cx+d=x^2-cx$ at $x=2$
$2c+d=4-2c$
$4c+d=4$

$$f(x)=\begin{cases} cx+d & \text{for } x\le 2 \\ x^2-cx & \text{for } x>2 \end{cases}$$

$c=2x-c$ at $x=2$
$c=4-c$
$2c=4$ $c=2$

25. Let f be the function defined above, where c and d are constants. If f is differentiable at $x=2$, what is the value of $c+d$?

$2+-4=-2$ $4(2)+d=4$
$d=-4$

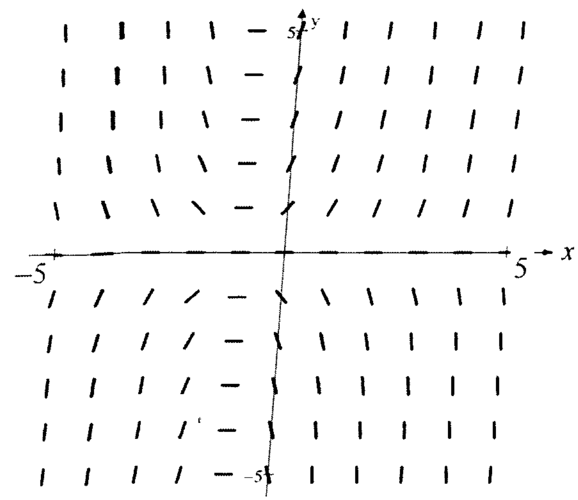
(A) –4 (B) –2 B (C) 0 (D) 2 (E) 4

26. What is the slope of the line tangent to the curve $y=\arctan(4x)$ at the point at which $x=\frac{1}{4}$?

$y'=\frac{1}{1+(4x)^2}\cdot 4$ $y'(\frac{1}{4})=\frac{1}{1+1^2}\cdot 4=\frac{4}{2}=2$

(A) 2 A (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) –2

0 at $x=-1$
0 at $y=0$



27. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx}=xy$

(B) $\frac{dy}{dx}=xy-y$ $y(x-1)$ 0 at $y=0$ and $x=1$

(C) $\frac{dy}{dx}=xy+y$ C $y(x+1)$ 0 at $y=0$ and $x=-1$

(D) $\frac{dy}{dx}=xy+x$

(E) $\frac{dy}{dx}=(x+1)^3$

28. Let f be a differentiable function such that $f(3)=15$, $f(6)=3$, $f'(3)=-8$, and $f'(6)=-2$. The function g is differentiable and $g(x)=f^{-1}(x)$ for all x. What is the value of $g'(3)$?

orig pt (3,15) inv (15,3)

(6,3) (3,6) using this set

$g'(3)=\frac{1}{f'(6)}=\frac{1}{-2}$

(A) $-\frac{1}{2}$ A

(B) $-\frac{1}{8}$

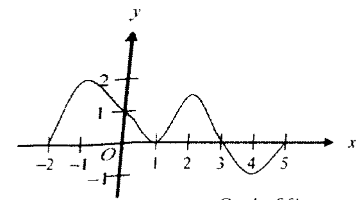
(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

(E) The value of $g'(3)$ cannot be determined from the information given.

End of Non-Calculator Multiple Choice

Multiple Choice Calculator



Graph of f'

increasing when $f'(x)>0$

76. The graph of f', the derivative f, is shown above for $-2 \le x \le 5$. On what intervals is f increasing?

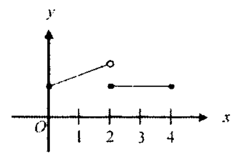
(A) $[-2, 1]$ only

(B) $[-2, 3]$ B

(C) $[3, 5]$ only

(D) $[0, 1.5]$ and $[3, 5]$

(E) $[-2, -1]$, $[1, 2]$, and $[4, 5]$



Graph of f

77. The figure above shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?

✓ I. $\lim_{x\to 2^-} f(x)$ exists. Yes

✓ II. $\lim_{x\to 2^+} f(x)$ exists. Yes

X III. $\lim_{x\to 2} f(x)$ exists. No, limit left ≠ limit right

(A) I only (B) II only (C) I and II only C (D) I and III only (E) I, II, and III

78. The first derivative of the function f is defined by $f'(x)=\sin(x^3-x)$ for $0 \le x \le 2$. On what interval(s) is f increasing? increasing when $f'(x) > 0$

Use calculator

(A) $1 \le x \le 1.445$

(B) $1 \le x \le 1.691$ B

(C) $1.445 \le x \le 1.875$

(D) $0.577 \le x \le 1.445$ and $1.875 \le x \le 2$

(E) $0 \le x \le 1$ and $1.691 \le x \le 2$

79. If $\int_{-5}^{2} f(x)\,dx = -17$ and $\int_{5}^{2} f(x)\,dx = -4$, what is the value of $\int_{-5}^{5} f(x)\,dx$? flip bounds

(A) −21 B (B) −13 (C) 0 (D) 13 (E) 21

$\int_{-5}^{2} f(x)dx + \int_{2}^{5} f(x)dx = -17+4 = -13$

80. The derivative of the function f is given by $f'(x)=x^2\cos(x^2)$. How many points of inflection does the graph of f have on the open interval $(-2, 2)$? $f''(x)$ changes sign

(A) One (B) Two (C) Three (D) Four E (E) Five

81. If $G(x)$ is an antiderivative for $f(x)$ and $G(2)=-7$, then $G(4)=$

old pos + change

(A) $f'(4)$

(B) $-7+f'(4)$ $G(4)=G(2)+\int_2^4 f(x)dx$

(C) $\int_2^4 f(t)\,dt$ $= -7+\int_2^4 f(x)dx$

(D) $\int_2^4 (-7+f(t))\,dt$

(E) $-7+\int_2^4 f(t)\,dt$ E

82. A particle moves along a straight line with velocity given by $v(t)=7-(1.01)^{-t^2}$ at time $t \ge 0$. What is the acceleration of the particle at time $t=3$? $V'(3) \approx .055$

(A) −0.914 B (B) 0.055 (C) 5.486 (D) 6.086 (E) 18.087

83. What is the area enclosed by the curves $y = x^3 - 8x^2 + 18x - 5$ (y_1) and $y = x + 5$ (y_2)?

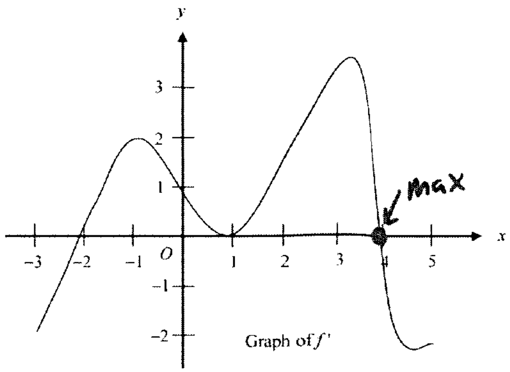
(A) 10.667

(B) 11.833 B $\int_1^2 (y_1-y_2)dx + \int_2^5 (y_2-y_1)dx$

(C) 14.583 Area ≈ 11.833

(D) 21.333

(E) 32



84. The graph of the derivative of a function f is shown in the figure above. The graph has horizontal tangent lines at $x=-1$, $x=1$, and $x=3$. At which of the following values of x does f have a relative maximum?

f' changes + to −

(A) −2 only

(B) 1 only

(C) 4 only C

(D) −1 and 3 only

(E) −2, 1, and 4

x	−4	−3	−2	−1
$f(x)$	0.75	−1.5	−2.25	−1.5
$f'(x)$	−3	−1.5	0	1.5

$\int_{-4}^{-1} f'(x)dx = f(x)\Big|_{-4}^{-1} = f(-1)-f(-4) = -1.5-.75 = -2.25$

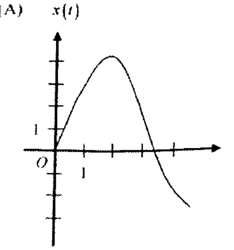
85. The table above gives values of a function f and its derivative at selected values of x. If f' is continuous on the interval $[-4, -1]$, what is the value of $\int_{-4}^{-1} f'(x)dx$? $= f(x)\Big|_{-4}^{-1}$

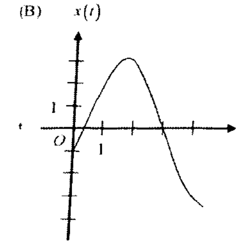
(A) −4.5 (B) −2.25 B (C) 0 (D) 2.25 (E) 4.5

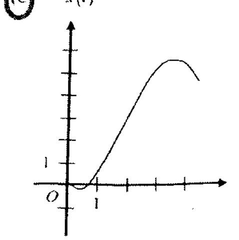
t	0	1	2	3	4
$v(t)$	−1	2	3	0	−4

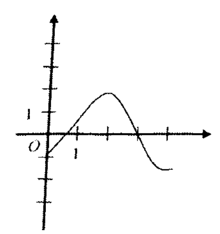
decrease increase increase Horiz. tan line decreasing

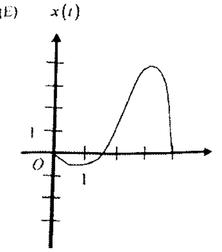
86. The table gives selected values of the velocity, $v(t)$, of a particle moving along the x-axis. At time $t=0$, the particle is at the origin. Which of the following could be the graph of the position, $x(t)$, of the particle for $0 \le t \le 4$?

(A) 

(B) 

C (C) 

(D) 

(E) 

87. An object traveling in a straight line has position $x(t)$ at time t. If the initial position is $x(0)=2$ and the velocity of the object is $v(t)=\sqrt[3]{1+t^2}$, what is the position of the object at time $t=3$?

D

(A) 0.431 (B) 2.154 (C) 4.512 (D) 6.512 (E) 17.408

$2+\int_0^3 v(t)\,dt \approx 6.512$

88. The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere? (The surface area S of a sphere with radius r is $S=4\pi r^2$)

C

(A) -108π (B) -72π (C) -48π (D) -24π (E) -16π

$S=4\pi r^2$ $\frac{dS}{dt}=8\pi r\frac{dr}{dt}$ $\frac{ds}{dt}=8\pi(3)(-2)=-48\pi$

89. The function f is continuous for $-2\le x\le 2$ and $f(-2)=f(2)=0$. If there is no c, where $-2<c<2$, for which $f'(c)=0$, which of the following statements must be true?

Means MVT does not apply so F(x) is not differentiable

(A) For $-2<k<2$, $f'(k)>0$.

(B) For $-2<k<2$, $f'(k)<0$.

(C) For $-2<k<2$, $f'(k)$ exists.

(D) For $-2<k<2$, $f'(k)$ exists, but f' is not continuous.

E

(E) For some k, where $-2<k<2$, $f'(k)$ does not exist.

90. The function f is continuous on the closed interval [2, 4] and twice differentiable on the open interval (2, 4). If $f'(3)=2$ and $f''(x)<0$ on the open interval (2,4), which of the following could be a table of values for f?

function is increasing at a decreasing rate

(A) A

x	$f(x)$
2	2.5
3	5
4	6.5

2.5, 1.5

(B)

x	$f(x)$
2	2.5
3	5
4	7

2.5, 2 Same as f'(3)

(C)

x	$f(x)$
2	3
3	5
4	6.5

2, 1.5 same rate & as f'(3)

(D) constant

x	$f(x)$
2	3
3	5
4	7

2, 2

(E) increasing rate

x	$f(x)$
2	3.5
3	5
4	7.5

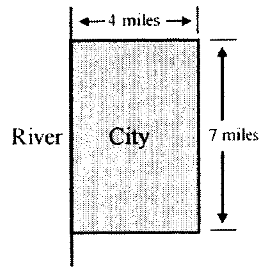
1.5, 2.5

91. What is the average value of $y=\frac{\cos x}{x^2+x+2}$ on the closed interval $[-1, 3]$?

C

(A) -0.085 (B) 0.090 (C) 0.183 (D) 0.244 (E) 0.732

$\frac{1}{3-(-1)}\int_{-1}^{3}\left(\frac{\cos x}{x^2+x+2}\right)dx \approx .183$



92. A city located beside a river has a rectangular boundary as shown in the figure above. The population density of the city at any point along a strip x miles from the river's edge is $f(x)$ persons per square mile. Which of the following expressions gives the population of the city?

(A) $\int_0^4 f(x)\,dx$

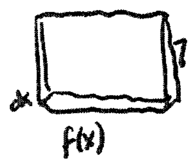
(B) $7\int_0^4 f(x)\,dx$

B

(C) $28\int_0^4 f(x)\,dx$

(D) $\int_0^7 f(x)\,dx$

(E) $4\int_0^7 f(x)\,dx$



$$\int_0^4 7f(x)\,dx = 7\int_0^4 f(x)\,dx$$