1990 AB3
Solution

(a) $A=\int_0^1 e^x-(x-1)^2\,dx$

$=\int_0^1 e^x-x^2+2x-1\,dx$

$=e^x\Big]_0^1-\frac{1}{3}(x-1)^3\Big]_0^1$

$=(e-1)-\frac{1}{3}=e-\frac{4}{3}$

(b) $V=\pi\int_0^1 e^{2x}-(x-1)^4\,dx$

$=\pi\left[\frac{e^{2x}}{2}\right]\Bigg|_0^1-\pi\left[\frac{1}{5}(x-1)^5\right]\Bigg|_0^1$

$=\pi\left[\left(\frac{e^2}{2}-\frac{1}{2}\right)-\frac{1}{5}\right]=\pi\left(\frac{e^2}{2}-\frac{7}{10}\right)$

or

$V=2\pi\int_0^1 y\left[1-\left(1-\sqrt{y}\right)\right]dy+2\pi\int_1^e y(1-\ln y)\,dy$

$=2\pi\cdot\frac{2}{5}y^{5/2}\Big|_0^1+2\pi\left[\frac{1}{2}y^2-\left(\frac{1}{2}y^2\ln y-\frac{1}{4}y^2\right)\right]\Bigg|_1^e$

$=\frac{4}{5}\pi+2\pi\left[\frac{1}{4}e^2-\frac{3}{4}\right]=\pi\left(\frac{e^2}{2}-\frac{7}{10}\right)$

(c) $V=2\pi\int_0^1 x\left[e^x-(x-1)^2\right]dx$

or

$V=\pi\int_0^1 1-\left(1-\sqrt{y}\right)^2dy+\pi\int_1^e 1-(\ln y)^2\,dy$

1991 BC3
Solution

(a) $\text{Area} = \int_0^{\pi/4} \cos x - \sin x \, dx$

$$= (\sin x + \cos x)\Big|_0^{\pi/4}$$

$$= \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - (0+1)$$

$$= \sqrt{2} - 1$$

(b) $V = \pi \int_0^{\pi/4} \cos^2 x - \sin^2 x \, dx$

$$= \pi \int_0^{\pi/4} \cos 2x \, dx$$

$$= \frac{\pi}{2} \sin 2x \Big|_0^{\pi/4}$$

$$= \frac{\pi}{2}(1-0) = \frac{\pi}{2}$$

(c) $V = \int_0^{\pi/4} (\cos x - \sin x)^2 \, dx$

$$= \int_0^{\pi/4} 1 - 2\sin x \cos x \, dx$$

$$= \left(x - \sin^2 x\right)\Big|_0^{\pi/4}$$

$$= \frac{\pi}{4} - \frac{1}{2} - (0-0)$$

$$= \frac{\pi}{4} - \frac{1}{2}$$

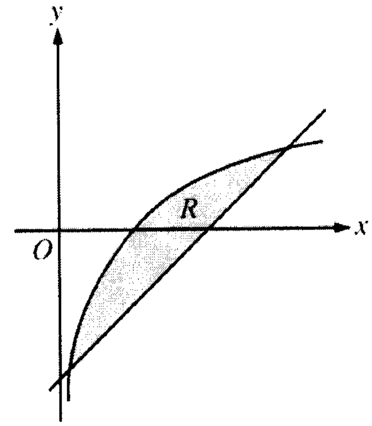
Question 1

Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

(c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.



$\ln(x) = x - 2$ when $x = 0.15859$ and 3.14619. Let $S = 0.15859$ and $T = 3.14619$	
(a) Area of $R = \int_S^T (\ln(x) - (x-2))\,dx = 1.949$	$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$
(b) Volume $= \pi \int_S^T \left((\ln(x) + 3)^2 - (x - 2 + 3)^2 \right) dx$ $= 34.198$ or 34.199	$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits, constant, and answer} \end{cases}$
(c) Volume $= \pi \int_{S-2}^{T-2} \left((y+2)^2 - \left(e^y\right)^2 \right) dy$	$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$