1996 AB2

2

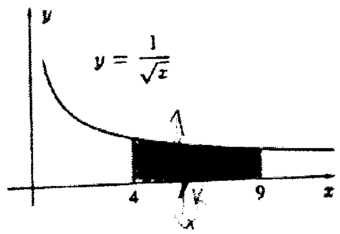
Solution

Scoring Scale

Points

(a) $\int_4^9 \frac{dx}{\sqrt{x}} = \boxed{2}$

$$3\begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \\ \quad 0/1 \text{ if integrand is incorrect} \end{cases}$$



(b) $\int_4^k \frac{dx}{\sqrt{x}} = 1$

$2\sqrt{x}\Big|_4^k = 1$

$2\sqrt{k} - 2\sqrt{4} = 1$ $\qquad 2\sqrt{K} - 4 = 1$

$\boxed{k = \frac{25}{4}}$ $\qquad 2\sqrt{K} = 5$

$\sqrt{K} = \frac{5}{2}$

$\left(\text{or } \int_k^9 \frac{dx}{\sqrt{x}} = 1 \text{ or } \int_4^k \frac{dx}{\sqrt{x}} = \int_k^9 \frac{dx}{\sqrt{x}}\right)$

$$3\begin{cases} 1: \int_4^k \frac{dx}{\sqrt{x}} \text{ or } \int_k^9 \frac{dx}{\sqrt{x}} \\ 1: \text{equation involving the two halves of } R \\ 1: \text{answer} \\ \quad 0/1 \text{ if answer from equation not} \\ \quad \text{involving relevant areas} \end{cases}$$

(c) volume $= \int_4^9 \left(\frac{1}{\sqrt{x}}\right)^2 dx$

$= \int_4^9 \frac{dx}{x} = \ln x\Big|_4^9 = \ln \frac{9}{4}$

$\boxed{\text{or } 0.811}$

$$3\begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \\ \quad 0/1 \text{ if integrand is incorrect} \end{cases}$$

3. The
in b

(a)

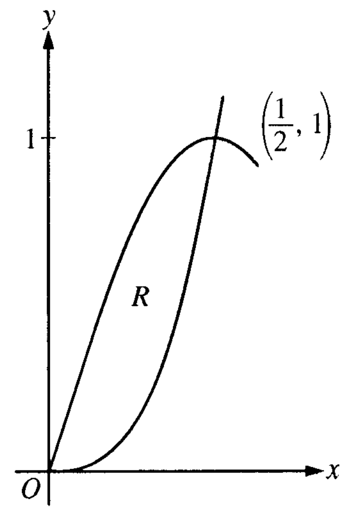
(b)

(c)

(d)

Question 3

Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.



(a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

(b) Find the area of R.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

(a) $f\left(\frac{1}{2}\right) = 1$

$f'(x) = 24x^2$, so $f'\left(\frac{1}{2}\right) = 6$

An equation for the tangent line is $y = 1 + 6\left(x - \frac{1}{2}\right)$.

$$\boxed{y - 1 = 6\left(x - \frac{1}{2}\right)}$$

$2 : \begin{cases} 1 : f'\left(\frac{1}{2}\right) \\ 1 : \text{answer} \end{cases}$

(b)
$$\begin{aligned} \text{Area} &= \int_0^{1/2} (g(x) - f(x))\,dx \\ &= \int_0^{1/2} \left(\sin(\pi x) - 8x^3\right) dx \\ &= \left[-\frac{1}{\pi}\cos(\pi x) - 2x^4\right]_{x=0}^{x=1/2} \\ &= \boxed{-\frac{1}{8} + \frac{1}{\pi}} \approx \boxed{.193} \end{aligned}$$

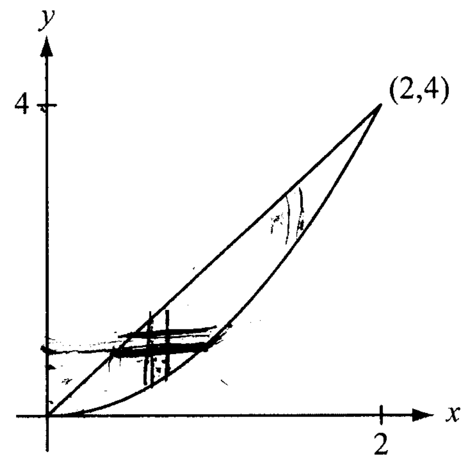
$4 : \begin{cases} 1 : \text{integrand} \\ 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(c)
$$\begin{aligned} &\pi \int_0^{1/2} \left((1 - f(x))^2 - (1 - g(x))^2\right) dx \\ &= \boxed{\pi \int_0^{1/2} \left(\left(1 - 8x^3\right)^2 - (1 - \sin(\pi x))^2\right) dx} \end{aligned}$$

$3 : \begin{cases} 1 : \text{limits and constant} \\ 2 : \text{integrand} \end{cases}$

1969 AB5
Solution

Intersection: $x^2 = 2x$ when $x = 0$ and $x = 2$



(a) Area $= \int_0^2 (2x - x^2)dx$

$$= x^2 - \frac{x^3}{3}\bigg|_0^2 = 4 - \frac{8}{3} = \boxed{\frac{4}{3}}$$

or

Area $= \int_0^4 \left(y^{1/2} - \frac{y}{2}\right)dy$

$$= \frac{2}{3}y^{3/2} - \frac{y^2}{4}\bigg|_0^4 = \frac{16}{3} - 4 = \frac{4}{3}$$

(b) <u>Shells</u>:

Volume $= \underline{2\pi\int_0^2 x(2x - x^2)dx} = 2\pi\left(\frac{2}{3}x^3 - \frac{x^4}{4}\right)\bigg|_0^2 = 2\pi\left(\frac{16}{3} - 4\right) = \boxed{\frac{8}{3}\pi}$

~~Disks~~ Washer

$y = x^2 \Rightarrow x = \pm\sqrt{y}$ $\quad y = 2x \Rightarrow x = \frac{y}{2}$

Volume $= \pi\int_0^4 \left(\left(y^{1/2}\right)^2 - \left(\frac{y}{2}\right)^2\right)dy = \pi\int_0^4 \left(y - \frac{y^2}{4}\right)dy$

$$= \pi\left(\frac{y^2}{2} - \frac{y^3}{12}\right)\bigg|_0^4 = \pi\left(8 - \frac{64}{12}\right) = \frac{8}{3}\pi$$

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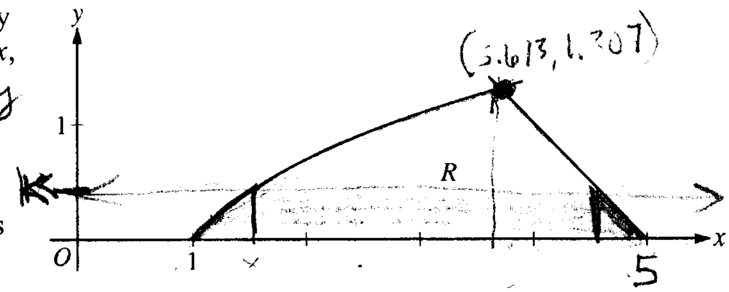
Question 2

Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

$x = e^y$ $\quad$ $x = 5 - y$



(a) Find the area of R.

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

$\ln x = 5 - x \Rightarrow x = 3.69344$

Therefore, the graphs of $y = \ln x$ and $y = 5 - x$ intersect in the first quadrant at the point $(A, B) = (3.69344, 1.30656)$.

(a) $\text{Area} = \int_0^B \left(5 - y - e^y\right) dy$
$= \boxed{2.986}$ (or 2.985)

OR

$\text{Area} = \int_1^A \ln x \, dx + \int_A^5 (5 - x)\, dx$
$= 2.986$ (or 2.985)

3 : { 1 : integrand; 1 : limits; 1 : answer }

(b) $\text{Volume} = \int_1^A (\ln x)^2 \, dx + \int_A^5 (5 - x)^2 \, dx$

3 : { 2 : integrands; 1 : expression for total volume }

(c) $\int_0^k \left(5 - y - e^y\right) dy = \frac{1}{2} \cdot 2.986 \left(\text{or } \frac{1}{2} \cdot 2.985\right)$

3 : { 1 : integrand; 1 : limits; 1 : equation }

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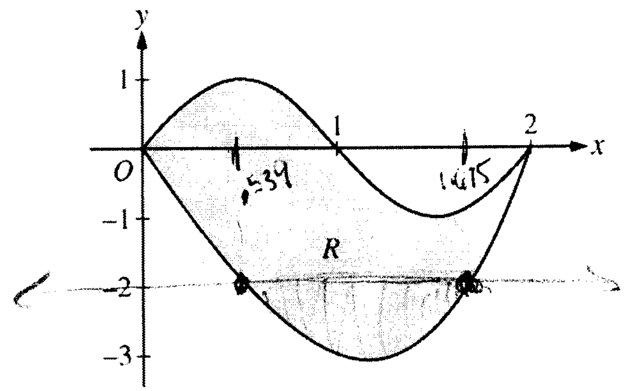
Question 1



Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

(a) Find the area of R.

(b) The horizontal line $y = -2$ splits the region R into two parts. Write, but do not evaluate, an integral expression for the area of the part of R that is below this horizontal line.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

(d) The region R models the surface of a small pond. At all points in R at a distance x from the y-axis, the depth of the water is given by $h(x) = 3 - x$. Find the volume of water in the pond.

(a) $\sin(\pi x) = x^3 - 4x$ at $x = 0$ and $x = 2$ Area $= \int_0^2 \left(\sin(\pi x) - \left(x^3 - 4x\right)\right) dx = \boxed{4}$	$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(b) $x^3 - 4x = -2$ at $r = 0.5391889$ and $s = 1.6751309$ The area of the stated region is $\int_r^s \left(-2 - \left(x^3 - 4x\right)\right) dx$	$2: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \end{cases}$
(c) Volume $= \int_0^2 \left(\sin(\pi x) - \left(x^3 - 4x\right)\right)^2 dx = \boxed{9.978}$	$2: \begin{cases} 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(d) Volume $= \int_0^2 (3 - x)\left(\sin(\pi x) - \left(x^3 - 4x\right)\right) dx = 8.369$ or $\boxed{8.370}$	$2: \begin{cases} 1: \text{integrand} \\ 1: \text{answer} \end{cases}$