Precalculus p.9 solutions

1. $\frac{\csc x}{\sin x} - \frac{\cot x}{\tan x} = \frac{\frac{1}{\sin x}}{\sin x} - \frac{\frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x}} = \frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x}$

$= \frac{1-\cos^2 x}{\sin^2 x} = \frac{\sin^2 x}{\sin^2 x} = 1$

3. $\frac{\cot x}{\csc^2 x - 1} = \frac{\cot x}{\cot^2 x} = \frac{1}{\cot x} = \tan x$

4. $\tan^2 x \cos^2 x + \cot^2 x \sin^2 x = \frac{\sin^2 x}{\cos^2 x} \cdot \cos^2 x + \frac{\cos^2 x}{\sin^2 x} \cdot \sin^2 x$

$= \sin^2 x + \cos^2 x = 1$

5. $\frac{\sec x - \cos x}{\tan x} = \frac{\frac{1}{\cos x} - \cos x}{\frac{\sin x}{\cos x}} = \frac{\frac{1-\cos^2 x}{\cos x}}{\frac{\sin x}{\cos x}} = \frac{1-\cos^2 x}{\cos x} \cdot \frac{\cos x}{\sin x}$

$= \frac{\sin^2 x}{\cos x} \cdot \frac{\cos x}{\sin x} = \sin x$

6. $\frac{1+\tan x}{\sin x} - \sec x = \frac{1+\frac{\sin x}{\cos x}}{\sin x} - \frac{1}{\cos x} = \frac{\cos x\left(1+\frac{\sin x}{\cos x}\right) - 1(\sin$

$= \frac{\cos x + \sin x - \sin x}{\sin x \cos x} = \frac{\cos x}{\sin x \cos x} = \frac{1}{\sin x} = \csc x$

7. $\sin x\left(\frac{\sin x}{1-\cos x} + \frac{1-\cos x}{\sin x}\right) = \sin x\left(\frac{\sin^2 x + 1 - 2\cos x + \cos^2 x}{(1-\cos x)\sin x}\right)$ (1)

$\sin x \cdot \frac{(2-2\cos x)}{(1-\cos x)\sin x} = \frac{(\sin x)(2)(1-\cos x)}{(1-\cos x)\sin x} = 2$

11. $(\sin x - \cos x)^2 + 2\sin x - \cos x$

$= \sin^2 x - 2\sin x\cos x + \cos^2 x + 2\sin x - \cos x$ ($\sin^2 x + \cos^2 x = 1$)

$= 1 - 2\sin x\cos x + 2\sin x - \cos x$

$= -\cos x - 2\sin x\cos x + 1 + 2\sin x$

$= -\cos x\,(1 + 2\sin x) + 1\,(1 + 2\sin x)$

$= (1 + 2\sin x)(-\cos x + 1)$

---

p. 10-11 solutions

(4) $\dfrac{\sin^2 t}{\tan^2 t} = \dfrac{\sin^2 t}{\frac{\sin^2 t}{\cos^2 t}} = \sin^2 t \cdot \dfrac{\cos^2 t}{\sin^2 t} = \cos^2 t$

(6) $\cos^2\beta - \sin^2\beta$

$= \cos^2\beta - (1 - \cos^2\beta) = \cos^2\beta - 1 + \cos^2\beta = 2\cos^2\beta - 1$

(9) $(1 + \sin x)(1 - \sin x)$

$= 1 - \sin x + \sin x - \sin^2 x = 1 - \sin^2 x = \cos^2 x$

(10) $\cot^2 y\,(\sec^2 y - 1)$

$= \cot^2 y\,(\tan^2 y) = \dfrac{\cos^2 y}{\sin^2 y} \cdot \dfrac{\sin^2 y}{\cos^2 y} = 1$

(24) $\dfrac{\cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right)} = \dfrac{\sin x}{\cos x} = \tan x$

(42) $\dfrac{1+\csc\theta}{\sec\theta} - \cot\theta$

$= \dfrac{1+\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}} - \dfrac{\cos\theta}{\sin\theta} = \dfrac{(\sin\theta+1)}{\sin\theta}\cdot\dfrac{(\cos\theta)}{1} - \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sin\theta\cos\theta+\cos\theta\cos\theta}{\sin\theta}$

$= \dfrac{\sin\theta\cos\theta}{\sin\theta} = \cos\theta$

(63) $\tan^3 x \cdot \sec^2 x - \tan^3 x$

$= \tan^3 x(\sec^2 x - 1) = \tan^3 x \cdot \tan^2 x = \tan^5 x$