



When you solve an inequality, a solution is any value that will make the statement true.

**Example 1** Solve each equation or inequality.

A.  $(x+3)(x^2+1)(x-4)^2 = 0$

$$\begin{aligned} x = -3 \quad x = 4 \quad x^2 + 1 = 0 \\ x^2 = -1 \\ x = \pm\sqrt{-1} = \pm i \end{aligned}$$

B.  $(x+3)(x^2+1)(x-4)^2 > 0$  pos

$$(-3, 4) \cup (4, \infty)$$

D.  $(x+3)(x^2+1)(x-4)^2 \geq 0$  pos zeros

$$[-3, \infty)$$

C.  $(x+3)(x^2+1)(x-4)^2 < 0$  neg

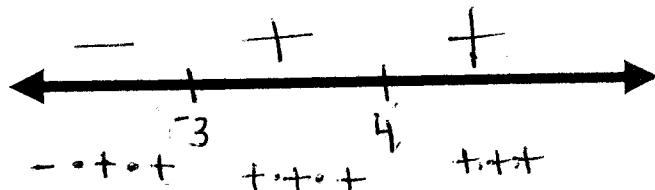
$$(-\infty, -3)$$

E.  $(x+3)(x^2+1)(x-4)^2 \leq 0$  neg zeros

$$(-\infty, -3] \cup [4, \infty]$$

**METHOD #1:**

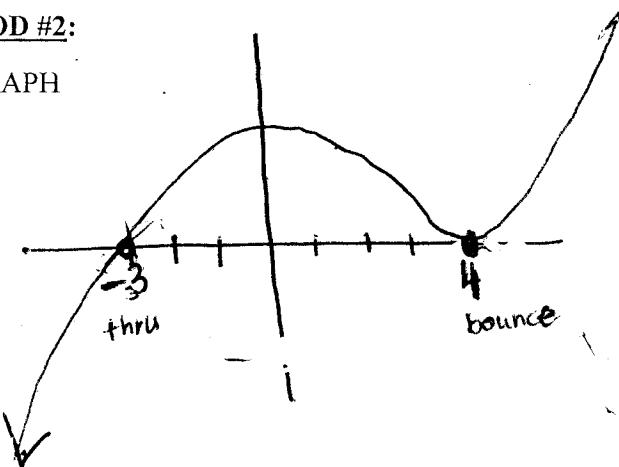
THE SIGN CHART



\*\*\*\*\*Note---ALL of the x-intercepts of the polynomial are placed on the sign chart\*\*\*\*\*

**METHOD #2:**

THE GRAPH



Example 2 Solve each inequality.

A.  $(x^2 + 7)(3x^2 + 1) \geq 0$  pos zeros

$$x^2 + 7 = 0 \quad 3x^2 + 1 = 0$$

$$x^2 = -7 \quad x^2 = -\frac{1}{3}$$

$$x = \pm\sqrt{-7} \quad x = \pm\sqrt{-\frac{1}{3}}$$

imaginary      imaginary

no real zeros

test  $x = 1$  + + + +

$$(-\infty, \infty)$$

C.  $(x^2 + x + 7)(3x + 1) \geq 0$  pos zeros

$$x^2 + x + 7 = 0 \quad 3x + 1 = 0$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(7)}}{2(1)} \quad x = -\frac{1}{3}$$

$$= \frac{-1 \pm \sqrt{-25}}{2} \quad - + +$$

complex

$$\left[-\frac{1}{3}, \infty\right)$$

E.  $2x^3 - 5x^2 \leq 18x - 45$

$$2x^3 - 5x^2 - 18x + 45 \leq 0$$

$$x^2(2x-5) - 9(2x-5) \leq 0$$

$$(2x-5)(x^2 - 9) \leq 0$$

$$(2x-5)(x+3)(x-3) \leq 0$$

neg zeros

zeros:  $\frac{5}{2}, -3, 3$

$$\begin{array}{c} - \\ - + + \\ - - - 3 - + - \frac{5}{2} + + - 3 + + \end{array}$$

G.  $x^2 - 2x \geq 15$

$$x^2 - 2x - 15 \geq 0$$

$$(x-5)(x+3) \geq 0$$

pos zeros

zeros:  $x = 5 \quad x = -3$

$$\begin{array}{c} + - + \\ + - + \\ -3 \quad 5 \end{array}$$

$$(-\infty, -3] \cup [5, \infty)$$

B.  $(x^2 + 7)(3x^2 + 1) \leq 0$  neg zeros

no soln.



D.  $(x^2 + x + 7)(3x + 1) > 0$  pos

$$\left(-\frac{1}{3}, \infty\right)$$

F.  $(x+2)(2x-3)(x-4) > 0$  pos

$2x^3 - 7x^2 - 10x + 24 > 0$

constant: 1, 2, 3, 4, 6, 8, 12, 24

I.C.: 1, 2

possible ratnl. roots:  $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24, \pm \frac{1}{2}, \pm \frac{3}{2}$

$$\begin{array}{r} -2 \\ \underline{-} \quad 2 \quad -7 \quad -10 \quad 24 \\ \quad \quad -4 \quad 22 \quad -24 \\ \hline \quad \quad 2 \quad 11 \quad 12 \quad 0 \end{array}$$

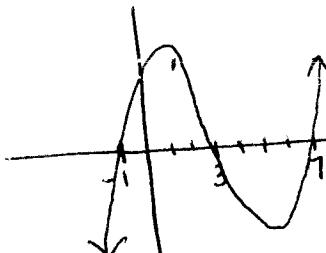
$$2x^2 - 11x + 12 = 0$$

$$(2x-3)(x-4) = 0$$

$$x = \frac{3}{2} \quad x = 4$$

H.  $x^3 - 9x^2 + 11x + 2 < 0$

graph on calc below x-axis  $(-2, \frac{3}{2}), (4, 10)$



$$(-\infty, -1) \cup (3, 7)$$



When you solve a rational inequality, you must include all of the zeros AND undefined values for the function on the sign chart.

Example 3 Solve each equation or inequality.

A.  $\frac{2x+1}{x^2+2x-3} = 0 \rightarrow (x+3)(x-1)$

zeros:  $2x+1=0$   
 $x = -\frac{1}{2}$

undefined:  $x = -3, 1$

B.  $\frac{2x+1}{x^2+2x-3} > 0$

$(-3, -\frac{1}{2}) \cup (1, \infty)$

$\begin{array}{c} - \\ \hline - & + & - & + \\ -3 & -\frac{1}{2} & 1 & \\ \hline + & + & + & + \end{array}$

C.  $\frac{2x+1}{x^2+2x-3} \leq 0$

$(-\infty, -3) \cup [-\frac{1}{2}, 1)$

Example 4 Solve each inequality.

A.  $\frac{5}{x+3} < -\frac{3}{x-1}$

$\frac{(x-1)}{(x-1)} \frac{5}{(x+3)} + \frac{3(x+3)}{(x-1)(x+3)} < 0$

LCD:  $(x+3)(x-1)$

$\frac{5x-5+3x+9}{(x+3)(x-1)} < 0$

$\frac{8x+4}{(x+3)(x-1)} < 0$

$\frac{4(2x+1)}{(x+3)(x-1)} < 0$

$(x+3)(x-1)$

zero:  $-\frac{1}{2}$

undef.:  $-3, 1$

B.  $\frac{5x+1}{2x^2} > \frac{9x+5}{4x^2+2x}$

$\frac{5x+1}{2x^2} - \frac{9x+5}{2x(2x+1)} > 0$

LCD:  $2x^2(2x+1)$

$\frac{(2x+1)(5x+1) - x(9x+5)}{2x^2(2x+1)} > 0$

$\frac{10x^2 + 7x + 1 - 9x^2 - 5x}{2x^2(2x+1)} > 0$

$\frac{x^2 + 2x + 1}{2x^2(2x+1)} > 0$

$\frac{(x+1)(x+1)}{2x^2(2x+1)} > 0$

zeros:  $-1, 0$

undef:  $0, -\frac{1}{2}$

$(-\infty, -3) \cup (-\frac{1}{2}, 1)$