

2.8----Solving Equations in One Variable

When MULTIPLYING BY x , or some expression involving an x (this includes "squaring both sides") then it is possible to find a solution that does NOT truly solve the equation when the answer is substituted back in. These solutions are referred to as extraneous solutions.

Ex1) Solve:

$$\frac{x}{1} + \frac{3}{x} = \frac{4}{1} \quad \text{LCD} = x$$

$$(x) \left(x + \frac{3}{x} \right) = (4)(x)$$

$$x^2 + \frac{3x}{x} = 4x$$

$$x^2 + 3 = 4x$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x-3=0 \quad x-1=0$$

$$\boxed{x=3} \quad \boxed{x=1}$$

STEP #1: Identify the L.C.D.

STEP #2: Multiply both sides by L.C.D.

(remember to DISTRIBUTE!!)

STEP #3: Simplify & Solve ☺

STEP #4: Check your answers !!!!!

$$3 + \frac{3}{3} \stackrel{?}{=} 4 \quad \checkmark \quad 1 + \frac{3}{1} \stackrel{?}{=} 4 \quad \checkmark$$

Ex2)
$$\frac{(x-4)}{1} \left(\frac{x}{1} + \frac{4}{x-4} \right) = (0)(x-4)$$
 LCD = $x-4$

$$x(x-4) + \frac{4(x-4)}{x-4} = 0$$

$$x^2 - 4x + 4 = 0$$

$$(x-2)(x-2) = 0$$

$$x-2=0$$

$$\boxed{x=2}$$

$$2 + \frac{4}{2-4} \stackrel{?}{=} 0$$

$$2 + \frac{-2}{0} \quad \checkmark$$

Ex3)
$$(x-1)(x-3) \left(\frac{2x}{x-1} + \frac{1}{x-3} \right) = \left(\frac{2}{x^2-4x+3} \right) (x-1)(x-3)$$

$$\text{LCD} = (x-1)(x-3)$$

$$\frac{2x(x-1)(x-3)}{x-1} + \frac{1(x-1)(x-3)}{x-3} = \frac{2(x-1)(x-3)}{(x-3)(x-1)}$$

$$2x^2 - 6x + x - 1 = 2$$

$$2x^2 - 5x - 3 = 0$$

$$(2x+1)(x-3) = 0$$

$$2x+1=0 \quad x-3=0$$

$$\boxed{x = -\frac{1}{2}} \quad x=3 \text{ extraneous}$$

$$\frac{-1}{-3} + \frac{1}{-2} = \frac{2}{\frac{1}{4} + 2 + 3}$$

$$\frac{\frac{2}{3} + -\frac{2}{7}}{\frac{14-6}{21}} = \frac{\frac{2}{21}}{\frac{21}{4}}$$

$$\text{Ex 4) } (3x-1)(2x+5) \left(\frac{2x+2}{2x+5} - \frac{x-4}{3x-1} \right) = \left(\frac{5x^2+18}{(3x-1)(2x+5)} \right) (3x-1)(2x+5)$$

$$\text{Ex 5) } t(t-4) \left(\frac{t+4}{t} + \frac{-4}{t-4} \right) = \left(\frac{-16}{t(t-4)} \right) (t(t-4))$$

$$\text{LCD} = (3x-1)(2x+5)$$

$$\text{LCD} = t(t-4)$$

$$(3x-1)(2x+2) - (2x+5)(x-4) = 5x^2 + 18$$

$$(t-4)(t+4) + -4t = -16$$

$$6x^2 + 6x - 2x - 2 - 2x^2 + 8x - 5x + 20 = 5x^2 + 18 \quad t^2 - 16 - 4t = -16$$

$$4x^2 + 7x + 18 = 5x^2 + 18$$

$$t^2 - 4t = 0$$

$$-x^2 + 7x = 0$$

$$\begin{array}{l} -x=0 \quad x-7=0 \\ \boxed{x=0} \quad \boxed{x=7} \end{array}$$

$$-x(x-7) = 0$$

$$t(t-4) = 0$$

Now You Try:

$$6) \frac{5x}{x-2} = 7 + \frac{10}{x-2}$$

$$(x+1)(x-1) \left(\frac{4x+1}{x+1} = \left(\frac{12}{x^2-1} + \frac{3}{1} \right) (x+1)(x-1) \right)$$

~~t=0~~ ~~t-4=0~~
~~t=4~~
no solution

$$\text{LCD} = (x+1)(x-1)$$

$$(x-1)(4x+1) = 12 + 3(x+1)(x-1)$$

$$4x^2 + x - 4x - 1 = 12 + 3x^2 - 3$$

$$x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

$$x-5=0 \quad x+2=0$$

$$\boxed{x=5} \quad \boxed{x=-2}$$

$$8) \frac{1}{x-2} + \frac{x-3}{7-x} = \frac{x+1}{-x^2+9x-14}$$