

The main objective is to ISOLATE the trig function(s) and ask “what angle on the unit circle yields this length or coordinate”. You must take into account the given interval—we are not restricted to the same intervals that we used for inverses.

Example 1 Solving on an interval: Find all values of x on the interval $[0, 2\pi)$.

A. $2\sin x + 1 = 0$

$$\begin{aligned} 2\sin x &= -1 \\ \sin x &= -\frac{1}{2} \\ x &= \sin^{-1}\left(-\frac{1}{2}\right) \\ x &= \boxed{\frac{7\pi}{6}, \frac{11\pi}{6}} \end{aligned}$$

B. $2\cos x = \sqrt{3}$

$$\begin{aligned} \cos x &= \frac{\sqrt{3}}{2} \\ x &= \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \\ x &= \boxed{\frac{\pi}{6}, \frac{11\pi}{6}} \end{aligned}$$

C. $\tan x = \sqrt{3}$

$$\begin{aligned} x &= \tan^{-1}(\sqrt{3}) \\ x &= \boxed{\frac{\pi}{3}, \frac{4\pi}{3}} \end{aligned}$$

If we wish to find the general solutions, then we have to take into account the periodic nature of the trig function. The functions with a period of 2π must add “ $2\pi k$ ” to include ALL of the solutions. The functions with a period of π must add “ πk ” to obtain ALL of the solutions.

Example 2 Finding general solutions: Find all solutions for each equation:

A. $\sqrt{2} + 2\cos x = 0$

$$\begin{aligned} 2\cos x &= -\sqrt{2} \\ \cos x &= -\frac{\sqrt{2}}{2} \\ x &= \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) \\ x &= \boxed{\frac{3\pi}{4} + 2\pi k, \frac{5\pi}{4} + 2\pi k} \end{aligned}$$

B. $4\cot \theta - 6 = -2$

$$\begin{aligned} 4\cot \theta &= 4 \\ \cot \theta &= 1 \\ \theta &= \cot^{-1}(1) \\ \theta &= \boxed{\frac{\pi}{4} + \pi k} \end{aligned}$$

C. $5 + \sin^2 x = 3$

$$\begin{aligned} \sin^2 x &= -2 \\ \sin x &= \pm \sqrt{-2} \\ &\text{no solution} \end{aligned}$$

Trigonometric equations in quadratic form: Use the same methods as you did in previous units (factoring, square root property, etc.)

Example 3 Find all values of x on the interval $[0, 2\pi)$.

A. $\cos^2 x \sin x - \sin x = 0$

$$\sin x (\cos^2 x - 1) = 0$$

$$\sin x (\cos x + 1)(\cos x - 1) = 0$$

$$\sin x = 0 \quad \cos x + 1 = 0 \quad \cos x - 1 = 0$$

$$x = \sin^{-1}(0) \quad x = \cos^{-1}(-1) \quad x = \cos^{-1}(1)$$

$$x = 0, \pi \quad x = \pi \quad x = 0$$

B. $\cos^2 x - 2\cos x = 3$

$$\cos^2 x - 2\cos x - 3 = 0$$

$$(\cos x - 3)(\cos x + 1) = 0$$

$$\cos x - 3 = 0 \quad \cos x + 1 = 0$$

$$x = \cos^{-1}(3) \quad x = \cos^{-1}(-1)$$

$$x = \pi$$

C. $2\sin^2 x + \sin x - 1 = 0$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$$2\sin x - 1 = 0 \quad \sin x + 1 = 0$$

$$x = \sin^{-1}\left(\frac{1}{2}\right) \quad x = \sin^{-1}(-1)$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6} \quad x = 3$$

$$\boxed{x = 0, \pi}$$

Using identities to solve trig equations: Equations may have 2 or more trig functions. Try factoring to see if the function can be separated. If not, use identities to rewrite the function into a single trig function or into a form that can be solved by factoring.

Example 4 Find all values of x on the interval $[0, 2\pi]$.

$$A. \cos^2 x + \cos x = \sin^2 x$$

$$\begin{aligned} \cos^2 x + \cos x - 1 &= 0 \\ 2\cos^2 x + \cos x - 1 &= 0 \\ (2\cos x - 1)(\cos x + 1) &= 0 \end{aligned}$$

$$\begin{aligned} 2\cos x - 1 &= 0 & \cos x + 1 &= 0 \\ \cos^{-1}\left(\frac{1}{2}\right) &= x & \cos^{-1}(-1) &= x \\ x = \frac{\pi}{3}, \frac{5\pi}{3} & & x = \pi & \end{aligned}$$

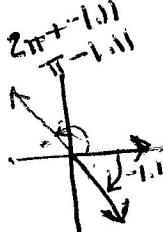
$$C. 2\cos^2 x + \sin x = 1$$

$$\begin{aligned} 2(1 - \sin^2 x) + \sin x &= 1 \\ 2 - 2\sin^2 x + \sin x &= 1 \\ 0 &= 2\sin^2 x - \sin x - 1 \\ 0 &= (\sin x + 1)(\sin x - 1) \\ 2\sin x + 1 &= 0 & \sin x - 1 &= 0 \end{aligned}$$

$$B. \tan x + \sec^2 x = 3$$

$$\begin{aligned} \tan x + 1 + \tan^2 x &= 3 \\ \tan^2 x + \tan x - 2 &= 0 \\ (\tan x + 2)(\tan x - 1) &= 0 \end{aligned}$$

$$\begin{aligned} \tan x + 2 &= 0 & \tan x - 1 &= 0 \\ x = \tan^{-1}(-2) & & x = \tan^{-1}(1) & \\ x = -1.11 & & x = \frac{\pi}{4} & \\ x = \frac{5.18}{2.03} & & x = \frac{\pi}{4} & \end{aligned}$$



$$\begin{aligned} x = \sin^{-1}\left(-\frac{1}{2}\right) & & x = \sin^{-1}(1) & \\ x = \frac{7\pi}{6}, \frac{11\pi}{6} & & x = \frac{\pi}{2} & \end{aligned}$$

Solving equations with multiple angles: The equations will not always have just "x" as the angle. You may have $3x$, $2x$, or $\frac{1}{2}x$. You must consider the interval you are solving over and how the period is changed.

Example 5 Find all values of x on the interval $[0, 2\pi]$.

$$A. 2\cos(4x) = -1$$

$$\begin{aligned} 0 \leq x < 2\pi & \\ 0 \leq 4x < 8\pi & \\ \cos(4x) = -\frac{1}{2} & \\ 4x = \cos^{-1}\left(-\frac{1}{2}\right) & \end{aligned}$$

$$4x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3}, \frac{20\pi}{3}, \frac{22\pi}{3}$$

$$x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{6}$$

$$C. \sin\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\frac{x}{2} = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\frac{x}{2} = \frac{\pi}{3}, \frac{2\pi}{3}$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$0 \leq x < 2\pi$$

$$0 \leq 4x < 8\pi$$

$$B. \csc(2x) = -1$$

$$\begin{aligned} \sin(2x) &= -1 \\ 2x &= \sin^{-1}(-1) \end{aligned}$$

$$0 \leq x < 2\pi$$

$$0 \leq 2x < 4\pi$$

$$2x = \frac{3\pi}{2}, \frac{7\pi}{2}$$

$$x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$\begin{aligned} 0 \leq x < 2\pi & \\ 0 \leq \frac{x}{2} < \pi & \end{aligned}$$

$$D. \tan(3x) = 1$$

$$3x = \tan^{-1}(1)$$

$$3x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \frac{17\pi}{4}, \frac{21\pi}{4}$$

$$0 \leq x < 2\pi$$

$$0 \leq 3x < 6\pi$$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{7\pi}{4}$$

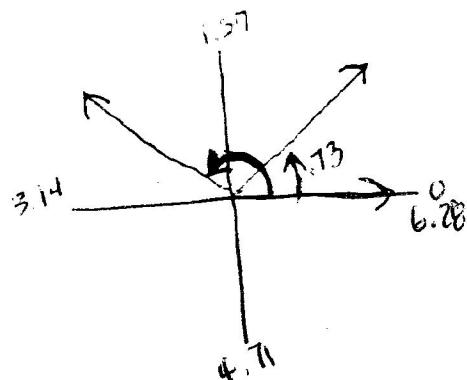
Solving equations with a calculator: Not all equations have "nice" solutions. You must consider reference angles when solving equations that require you to find angles on the calculator.

Example 6 Find all values of x on the interval $[0, 2\pi)$.

A. $3 \sin x = 2$

$$x = \sin^{-1} \left(\frac{2}{3} \right)$$

$$x = .73, 2.41$$



B. $\tan x = -4$

$$x = \tan^{-1}(-4)$$

$$x = -1.33$$

$$x = 4.96, 1.82$$

