Notes (Section 5.3)- Sum & Difference Formulas

$cos(A+B) = cosAcosB - sinAsinB$	$sin(A+B) = sinAcosB + cosAsinB$
$cos(A-B) = cosAcosB + sinAsinB$	$sin(A-B) = sinAcosB - cosAsinB$
$tan(A+B) = \frac{tanA+tanB}{1-tanAtanB}$	$tan(A-B) = \frac{tanA-tanB}{1+tanAtanB}$

Use sum and difference formulas to prove other trigonometric identities:

1) $\cos(\pi/2 - x) = \sin x$ 2) $\sin(\pi/2 - x) = \cos x$ 3) $\sin(x+\pi) = -\sin x$ 4) $\cos\left(x + \frac{3\pi}{2}\right) = \sin x$

$\cos\frac{\pi}{2}\cdot\cos x + \sin\frac{\pi}{2}\cdot\sin x$ $\sin\frac{\pi}{2}\cdot\cos x - \cos\frac{\pi}{2}\cdot\sin x$ $\sin x\cdot\cos\pi + \cos x\cdot\sin\pi$ $\cos x\cdot\cos\frac{3\pi}{2} - \sin x\cdot\sin\frac{3\pi}{2}$

$0\cdot\cos x + 1\cdot\sin x$ $1\cdot\cos x - \cancel{0\cdot\sin x}$ $\sin x\cdot -1 + \cancel{\cos x\cdot 0}$ $\cos x\cdot 0 - \sin x\cdot -1$

$\sin x$ $\cos x$ $-\sin x$ $\sin x$

Use the sum or difference identity to find the exact value (without using a calculator):

5) $\cos 75^{\circ}$ 6) $\sin 75^{\circ}$ 7) $\cos\left(\frac{\pi}{12}\right)$ 8) $\tan 255^{\circ}$ 9) $\tan\left(\frac{5\pi}{12}\right)$

Use the sum and difference formulas to write each of the following expressions as the sine or cosine of a single angle.

10) sin 22° cos 13° + cos 22° sin 13°

$\sin(22^{\circ} + 13^{\circ})$

$\sin 35^{\circ}$

11) cos (π/3) cos (π/4) – sin (π/3) sin (π/4)

$\cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \cos\left(\frac{4\pi}{12} + \frac{3\pi}{12}\right) = \cos\left(\frac{7\pi}{12}\right)$

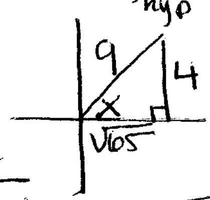
Use sum and difference formulas to evaluate the following:

12) Find the value of $cos(x-y)$ if $0 < x < \frac{\pi}{2}$, $0 < y < \frac{\pi}{2}$, $sinx = \frac{4}{9}$ (opp / hyp), $sin y = \frac{1}{4}$

$\cos(x-y)$

$\cos x\cos y + \sin x\sin y$

$\frac{\sqrt{65}}{9}\cdot\frac{\sqrt{15}}{4} + \frac{4}{9}\cdot\frac{1}{4} = \frac{\sqrt{975}}{36} + \frac{4}{36}$

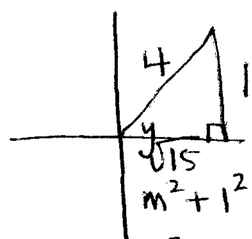


$m^2 + 4^2 = 9^2$

$m^2 + 16 = 81$

$m^2 = 65$

$m = \sqrt{65}$



$m^2 + 1^2 = 4^2$

$m^2 + 1 = 16$

$m^2 = 15$

$m = \sqrt{15}$

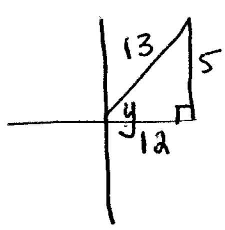
$\boxed{\frac{\sqrt{975}+4}{36}}$

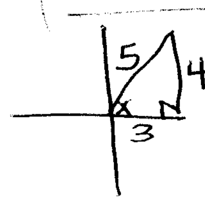
13) Find the value of $sin(x+y)$ if $0 < x < \frac{\pi}{2}$, $0 < y < \frac{\pi}{2}$, $sinx = \frac{4}{5}$, $sin y = \frac{5}{13}$

$\sin(x+y) = \sin x\cos y + \cos x\sin y$

$= \frac{4}{5}\cdot\frac{12}{13} + \frac{3}{5}\cdot\frac{5}{13}$

$= \frac{48}{65} + \frac{15}{65} = \boxed{\frac{63}{65}}$





5) $\cos 75^\circ$

$\cos(45^\circ + 30^\circ)$

$\cos 45^\circ \cdot \cos 30^\circ - \sin 45^\circ \cdot \sin 30^\circ$

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$\frac{\sqrt{6} - \sqrt{2}}{4}$

6) $\sin 75^\circ$

$\sin(45^\circ + 30^\circ)$

$\sin 45^\circ \cos 30^\circ + \sin 30^\circ \cdot \cos 45^\circ$

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$\frac{\sqrt{6} + \sqrt{2}}{4}$

7) $\cos\left(\frac{\pi}{12}\right)$

$\frac{\pi}{3} - \frac{\pi}{4}$

$\frac{4\pi - 3\pi}{12} = \frac{\pi}{12}$

$\frac{\pi}{12} \cdot \frac{180^\circ}{\pi} = 15^\circ$

$\frac{\pi}{4} + \frac{\pi}{6} = \frac{3\pi + 2\pi}{12} = \frac{5\pi}{12}$

$\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$\cos\frac{\pi}{3} \cdot \cos\frac{\pi}{4} + \sin\frac{\pi}{3} \cdot \sin\frac{\pi}{4}$

$\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$\frac{\sqrt{2} + \sqrt{6}}{4}$

9) $\tan\left(\frac{5\pi}{12}\right)$

$\tan\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$\frac{\tan\frac{\pi}{4} + \tan\frac{\pi}{6}}{1 - \tan\frac{\pi}{4} \cdot \tan\frac{\pi}{6}}$

$\frac{1 + \frac{\sqrt{3}}{3}}{1 - 1 \cdot \frac{\sqrt{3}}{3}} = \frac{3 + \sqrt{3}}{3 - \sqrt{3}}$

8) $\tan 255^\circ$

$\tan(225^\circ + 30^\circ)$

$\frac{\tan 225^\circ + \tan 30^\circ}{1 - \tan 225^\circ \cdot \tan 30^\circ}$

$\frac{\frac{1 \cdot 3}{1 \cdot 3} + \frac{\sqrt{3}}{3}}{\frac{1 \cdot 3}{1 \cdot 3} - 1 \cdot \frac{\sqrt{3}}{3}} = \frac{\frac{3 + \sqrt{3}}{3}}{\frac{3 - \sqrt{3}}{3}}$

$\frac{3 + \sqrt{3}}{\cancel{3}} \cdot \frac{\cancel{3}}{3 - \sqrt{3}} = \boxed{\frac{3 + \sqrt{3}}{3 - \sqrt{3}}}$