Find the Maclaurin polynomial of degree n for the function.

1) $f(x) = \sin x, \quad n = 5$

$x - \frac{1}{6}x^3 + \frac{1}{120}x^5$

2) $f(x) = xe^x, \quad n = 4$

$x + x^2 + \frac{1}{2}x^3 + \frac{1}{6}x^4$

3) $f(x) = \frac{1}{x+1}, \quad n = 4$

$1 - x + x^2 - x^3 + x^4$

Find the 4th Taylor polynomial centered at $c = 1$ with n = 4.

4) $f(x) = \frac{1}{x}$

$1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4$

5) $f(x) = \sqrt{x}$

$1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3 - \frac{5}{128}(x-1)^4$

6) $f(x) = \ln x$

$x - 1 + \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4$

7) Numerical and Graphical Approximations

a) Use the Maclaurin polynomials $P_1(x)$, $P_3(x)$, and $P_5(x)$, for $f(x) = \sin x$ to complete the table. $P_1(x) = x$ $\quad P_3(x) = x - \frac{1}{6}x^3$ $\quad P_5(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5$

x	0	0.25	0.50	0.75	1.00
$\sin x$	0	0.2474	0.4794	0.6816	0.8415
$P_1(x)$	0	.25	.50	.75	1
$P_3(x)$	0	.2474	.4792	.6797	.8333
$P_5(x)$	0	.2474	.4794	.6817	.8417

b) Use a calculator to graph $f(x) = \sin x$ and the Maclaurin polynomials in part (a).

c) Describe the change in accuracy of a polynomial approximation as the distance from the point where the polynomial is centered increases.

Farther away from the center ⇒ less accurate approximation

8) Numerical and Graphical Approximations

a) Use the Taylor polynomials $P_1(x)$, $P_2(x)$, and $P_4(x)$, for $f(x) = \ln x$ centered at $c = 1$ to complete the table. $P_1(x) = x - 1$ $\quad P_2(x) = x - 1 - \frac{1}{2}(x-1)^2$ $\quad P_4(x) = x - 1 - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4$

x	1.00	1.25	1.50	1.75	2.00
$\ln x$	0	0.2231	0.4055	0.5596	0.6931
$P_1(x)$	0	.25	.5	.75	1
$P_2(x)$	0	.2188	.3750	.4688	.5
$P_4(x)$	0	.2230	.4010	.5302	.5833

b) Use a calculator to graph $f(x) = \ln x$ and the Taylor polynomials in part (a).

c) Describe the change in accuracy of polynomial approximations as the degree increases.

as degree increases ⇒ the approximation gets closer to the actual value