

NOTES-- Transformations

Transformations of the graphs of functions

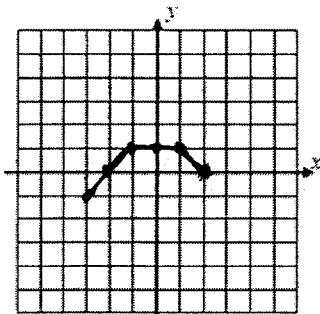
| | |
|------------|---|
| $f(x) + c$ | shift $f(x)$ up c units |
| $f(x) - c$ | shift $f(x)$ down c units |
| $f(x + c)$ | shift $f(x)$ left c units |
| $f(x - c)$ | shift $f(x)$ right c units |
| $f(-x)$ | reflect $f(x)$ about the y -axis |
| $-f(x)$ | reflect $f(x)$ about the x -axis |
| $cf(x)$ | When $0 < c < 1$ – vertical shrinking of $f(x)$ When $c > 1$ – vertical stretching of $f(x)$ Multiply the y values by c |
| $f(cx)$ | When $0 < c < 1$ – horizontal stretching of $f(x)$ When $c > 1$ – horizontal shrinking of $f(x)$ Divide the x values by c |

Order of function transformations

- Horizontal shifts
- Horizontal stretch/compression
- Reflection over y -axis
- Vertical stretch/compression
- Reflection over x -axis
- Vertical shifts

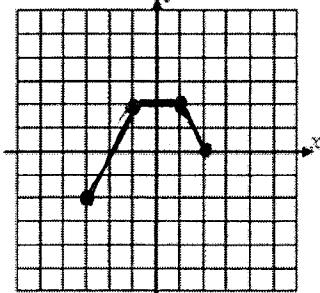
Example 1 Graph each transformation.

$$y = f(x)$$



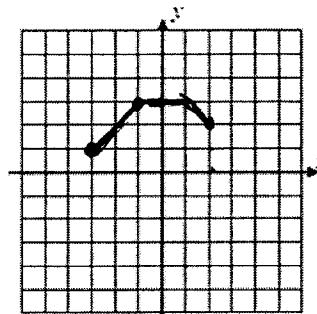
$$y = 2f(x)$$

vertical stretch by a factor of 2
* y's by 2



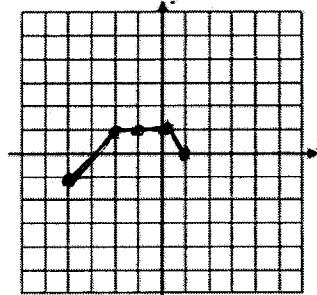
$$y = f(x) + 2$$

shift up 2



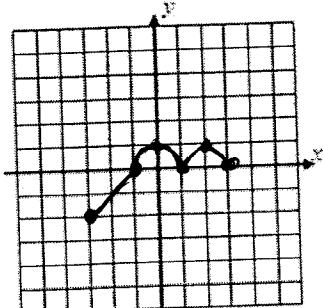
$$y = f(x + 1)$$

shift left 1

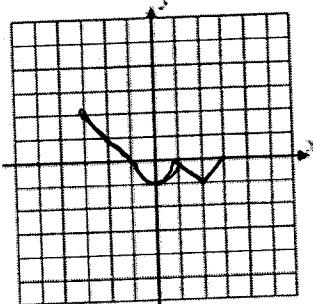


Example 2 Graph each transformation.

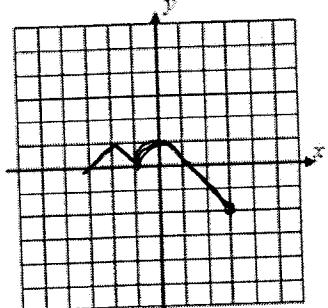
$$y = f(x)$$



$$y = -f(x) \text{ reflect over } x\text{-axis}$$

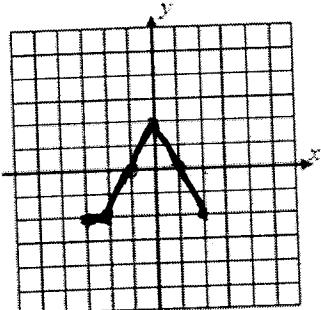


$$y = f(-x) \text{ reflect over } y\text{-axis}$$



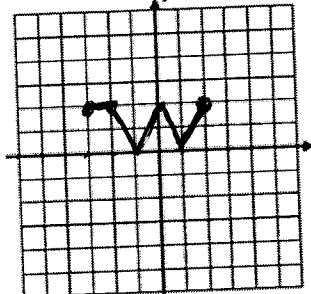
Example 3 Graph each transformation.

$$y = f(x)$$

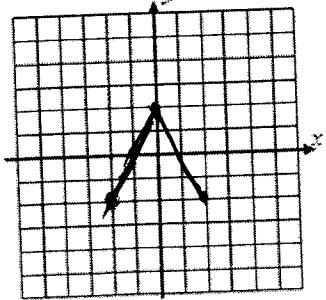


$$y = |f(x)| \text{ partial reflection over the } x\text{-axis}$$

y -coordinates become 0 or pos.



$$y = f(|x|) \text{ partial reflection over the } y\text{-axis}$$



Example 4 Given $f(x) = |x|$, write an equation reflecting the described transformations.

A. reflect over the x-axis, shift down 5

$$y = -|x| - 5$$

B. reflect over the y-axis, stretch vertically by a factor of 8, shift up 3 units

$$y = 8|-x| + 3$$

C. horizontally compress by a factor of $\frac{1}{2}$, reflect over the x-axis, shift down 10

$$y = -|2x| - 10$$

Example 5 Describe the transformations on the parent function.

A. $y = -2(x-5)^3 + 1$ parent $y = x^3$

shift right 5

vertical stretch by a factor of 2

reflect about x-axis

shift up 1

B. $y = 2^{-x} - 4$ parent $y = 2^x$

reflect over y-axis

shift down 4

C. $y = -\frac{3}{x} + 8$ parent $y = \frac{1}{x}$

vertical stretch by a factor of 3

reflect over x-axis

shift up 8

D. $y = -\frac{1}{2}\sqrt{x-7}$

parent $y = \sqrt{x}$

shift right 7

vertical shrink by a factor of $\frac{1}{2}$

reflect over x-axis