In Trigonometry we look at an angle in terms of a rotating ray. The beginning position of the ray is called the initial side of the angle.

The ray is rotated about its end point called the vertex and the final position of the ray is called the terminal side of the angle.

The degree of an angle is a number that describes the amount of rotation from the initial side to the terminal side of the angle.

Positive angles are generated by counterclockwise rotations

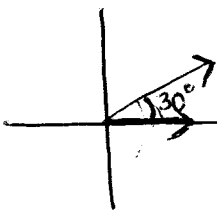
Negative angles are generated by clockwise rotations

***NOTE: Typically angles are drawn in STANDARD POSITION with vertex at the origin & initial side on the positive x-axis.

Because it is possible for two angles to have the same initial side and terminal side but different angle measures we refer to these angles as coterminal angles.

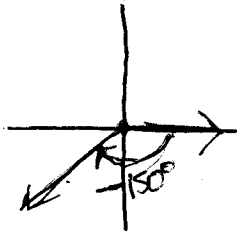
Ex 1) Find 2 positive and 2 negative angles that are co-terminal with the given angle.

a) 30°



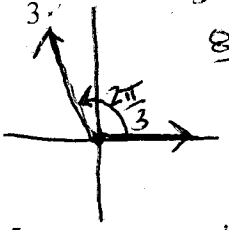
390°
750°
−330°
−690°

b) −150°



210°
570°
−510°
−870°

c) $\frac{2\pi}{3}$



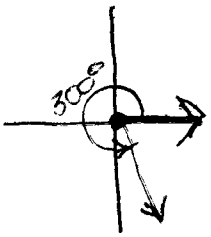
$\frac{2\pi}{3} + 2\pi = \frac{8\pi}{3}$

$\frac{8\pi}{3} + 2\pi = \frac{14\pi}{3}$

$\frac{2\pi}{3} - 2\pi = -\frac{4}{3}\pi$

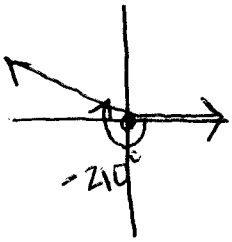
$-\frac{4\pi}{3} - 2\pi = -\frac{10\pi}{3}$

d) 300°



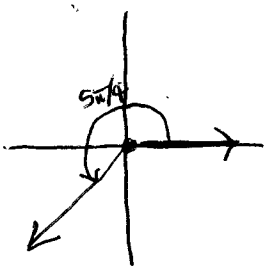
660°
1020°
−60°
−420°

e) −210°



150°
510°
−570°
−930°

f) $\frac{5\pi}{4}$

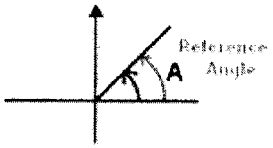


$\frac{13\pi}{4}, \frac{21\pi}{4}$

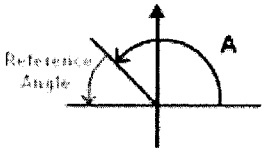
$-\frac{3}{4}\pi, -\frac{11}{4}\pi$

reference angle -- the angle that the given angle makes with the *x*-axis. Regardless of where the angle ends (that is, regardless of the location of the terminal side of the angle), the reference angle measures the closest distance of that terminal side to the *x*-axis.

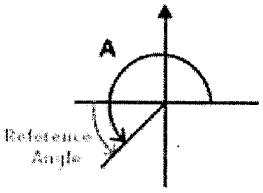
acute



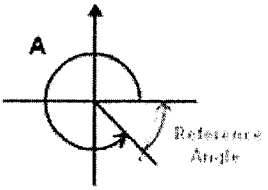
1st Quadrant



2nd Quadrant

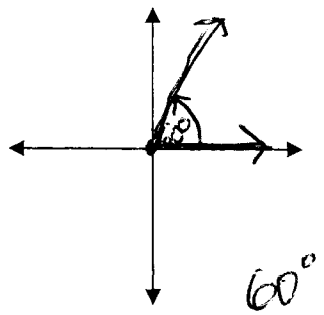
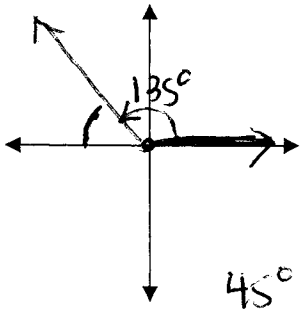
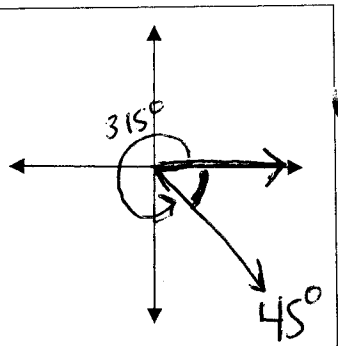
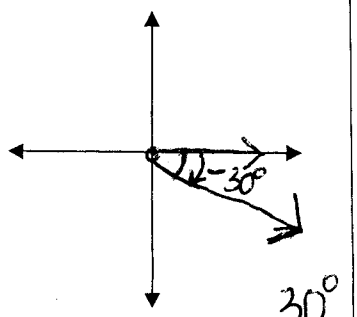
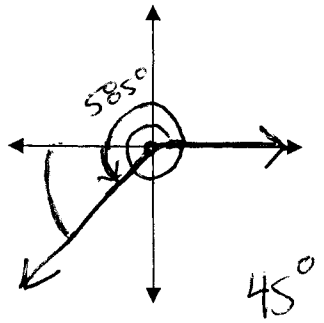
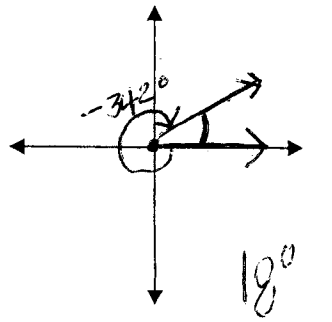


3rd Quadrant

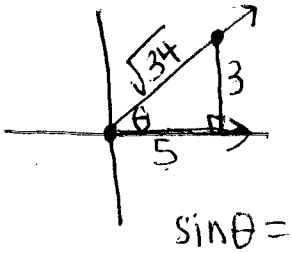


4th Quadrant

Ex 2) Draw each angle in standard position, and find its reference angle.

θ = 60° 	θ = 135° 	θ = 315° 
θ = -30° 	θ = 585° 	θ = -342° 

Ex 3) Let θ be the acute angle in standard position whose terminal side contains (5, 3). Find the 6 trig functions of θ.



$5^2 + 3^2 = h^2$

$34 = h^2$

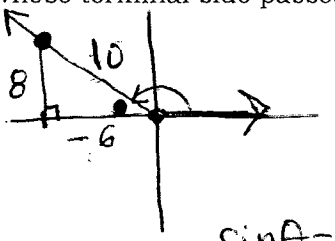
$h = \sqrt{34}$

$\sin\theta = \frac{3}{\sqrt{34}}$ $\csc\theta = \frac{\sqrt{34}}{3}$

$\cos\theta = \frac{5}{\sqrt{34}}$ $\sec\theta = \frac{\sqrt{34}}{5}$

$\tan\theta = \frac{3}{5}$ $\cot\theta = \frac{5}{3}$

Ex 4) Find the six trig functions of an angle whose terminal side passes through (-6, 8)



$(-6)^2 + 8^2 = h^2$

$100 = h^2$

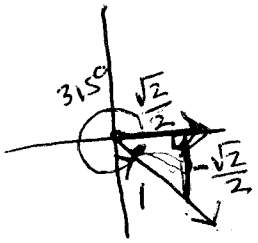
$h = 10$

$\sin\theta = \frac{8}{10} = \frac{4}{5}$ $\csc\theta = \frac{5}{4}$

$\cos\theta = \frac{-6}{10} = \frac{-3}{5}$ $\sec\theta = -\frac{5}{3}$

$\tan\theta = \frac{8}{-6} = -\frac{4}{3}$ $\cot\theta = -\frac{3}{4}$

Ex 5) Find the six trig functions of 315°



$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\sin 315° = -\frac{\sqrt{2}}{2}$

$\cos 315° = \frac{\sqrt{2}}{2}$

$\tan 315° = -1$

$\csc 315° = -\frac{2}{\sqrt{2}}$

$\sec 315° = \frac{2}{\sqrt{2}}$

$\cot 315° = -1$

Ex 6) Find each of the following:

a) $\sin(-210°)$ $\frac{1}{2}$

b) $\tan(5\pi/3)$ $\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\frac{\sqrt{3}}{2} \cdot \frac{2}{1} = -\sqrt{3}$

c) $\sec(-3\pi/4)$ $\frac{-2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-2\sqrt{2}}{2} = -\sqrt{2}$

Angles whose terminal sides lie along one of the coordinate axes are called quadrantal angles.

Ex 7) Find each of the following, if it exists:

a) $\sin(-270°)$ $(0,1)$ 1

b) $\tan 3\pi$ $(-1,0)$ $\frac{0}{-1} = 0$

c) $\sec(11\pi/2)$ $(0,-1)$ $\frac{1}{0}$ undefined

d) $\cos(180°)$ $(-1,0)$ -1

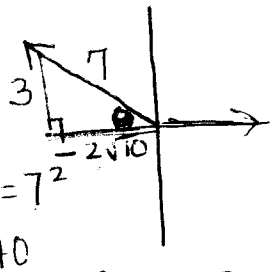
e) $\cot(0°)$ $(1,0)$ $\frac{1}{0}$ undefined

f) $\csc(-\pi/2)$ $(0,-1)$ -1

Ex 8) Find $\cos\theta$ and $\tan\theta$ using the given information to construct a reference angle.

a) $\sin\theta = 3/7$, $\tan\theta < 0$ → 2nd quad

S | A
T | C



$x^2 + 3^2 = 7^2$

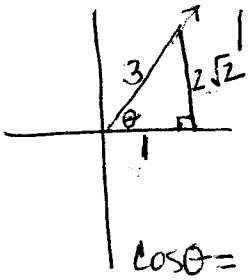
$x^2 = 40$

$x = -\sqrt{40} = -2\sqrt{10}$

$\cos\theta = \frac{-2\sqrt{10}}{7}$

$\tan\theta = \frac{3}{-2\sqrt{10}}$

b) $\sec\theta = \frac{3}{1}$, $\sin\theta > 0$ → 1st quad



$1^2 + y^2 = 3^2$

$y^2 = 8$

$y = \sqrt{8} = 2\sqrt{2}$

$\cos\theta = \frac{1}{3}$

$\tan\theta = 2\sqrt{2}$

c) $\cot\theta$ is undefined, $\sec\theta < 0$ → quadrantal

$\frac{x}{y} \leftarrow 0$

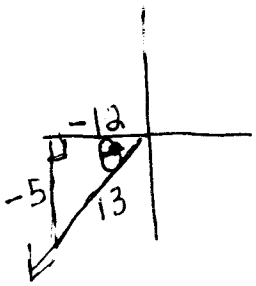
$(-1,0)$

$\cos\theta = -1$

$\tan\theta = \frac{0}{-1} = 0$

Ex 9) Find $\sin\theta$ and $\cot\theta$ using the given information to construct a reference angle.

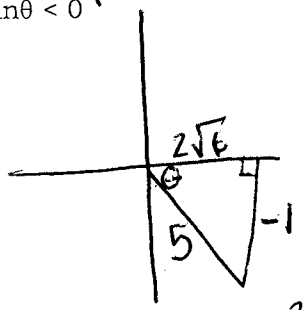
d) $\cos\theta = -12/13$, $\sin\theta < 0$



$\sin\theta = \frac{-5}{13}$

$\cot\theta = \frac{-12}{-5} = \frac{12}{5}$

e) $\csc\theta = \frac{-5}{1}$, $\tan\theta < 0$



$x^2 + (-1)^2 = 5^2$

$x^2 = 24$

$x = \sqrt{24} = 2\sqrt{6}$

$\sin\theta = -\frac{1}{5}$

$\cot\theta = -2\sqrt{6}$

f) $\tan\theta$ is undefined, $\sin\theta > 0$ → quadrantal

$\frac{y}{x} \leftarrow 0$

$(0,1)$

$\sin\theta = 1$

$\cot\theta = \frac{0}{1} = 0$