

When proving an Identity, you must show that the sides of the identity are equivalent. Begin with one side of the equation (typically the more complicated side). Use trig identities and algebra to write steps. Continue writing steps (your work must be logical and clear) until you get the other side of the equation.

Hints:

- Start with the more complicated side
- Work on one side only (you are rewriting and simplifying a side—not solving an equation!)
- Know your trig identities!
- If you're stuck, change to sines and cosines and look for ways to simplify
- Use Algebra! – combine fractions, factor, distribute, etc
- If it's taking a long time to get there, don't give up! The goal is to get there, not to have the shortest steps to get there!

Prove each identity.

1)  $\csc \theta \tan \theta = \sec \theta$

$$\frac{1}{\sin \theta} \cdot \frac{\sin \theta}{\cos \theta} = \frac{1}{\cos \theta} = \sec \theta$$

3)  $\frac{\cos x - 2 \sin x \cos x}{\cos^2 x - \sin^2 x + \sin x - 1} = \cot x$

$$\begin{aligned} \frac{\cos x (1 - 2 \sin x)}{1 - \sin^2 x - \sin^2 x + \sin x - 1} &= \frac{\cos x (1 - 2 \sin x)}{\sin x - 2 \sin^2 x} \\ &= \frac{\cos x (1 - 2 \sin x)}{\sin x (1 - 2 \sin x)} = \frac{\cos x}{\sin x} = \cot x \end{aligned}$$

5)  $\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} = 2 \sec x$

$$\begin{aligned} \frac{\cos^2 x + 1 + 2 \sin x + \sin^2 x}{(1 + \sin x) \cos x} &= \frac{2 + 2 \sin x}{(1 + \sin x) \cos x} \\ &= \frac{2(1 + \sin x)}{(1 + \sin x) \cos x} = \frac{2}{\cos x} = 2 \sec x \end{aligned}$$

7)  $\frac{\cos x}{1 - \sin x} = \frac{1 + \sin x}{\cos x}$

$$\begin{aligned} \frac{\cos x}{1 - \sin x} \cdot \frac{(1 + \sin x)}{(1 + \sin x)} &= \frac{\cos x (1 + \sin x)}{1 - \sin^2 x} \\ &= \frac{\cos x (1 + \sin x)}{\cos^2 x} = \frac{1 + \sin x}{\cos x} \end{aligned}$$

2)  $\tan x + \cot x = \sec x \csc x$

$$\begin{aligned} \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} &= \frac{\sin^2 x + \cos^2 x}{\cos x \sin x} = \frac{1}{\cos x \sin x} \\ &= \frac{1}{\cos x} \cdot \frac{1}{\sin x} = \sec x \cdot \csc x \end{aligned}$$

4)  $\cot \theta + \frac{1 - 2 \cos^2 \theta}{\sin \theta \cos \theta} = \tan \theta$

$$\begin{aligned} \frac{\cos \theta}{\sin \theta} + \frac{1 - 2 \cos^2 \theta}{\sin \theta \cos \theta} &= \frac{\cos^2 \theta + 1 - 2 \cos^2 \theta}{\sin \theta \cos \theta} \\ &= \frac{1 - \cos^2 \theta}{\sin \theta \cos \theta} = \frac{\sin^2 \theta}{\sin \theta \cos \theta} = \frac{\sin \theta}{\cos \theta} = \tan \theta \end{aligned}$$

6)  $\frac{\tan x - \cot x}{\tan x + \cot x} = \sin^2 x - \cos^2 x$

$$\begin{aligned} \frac{\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} &= \frac{\frac{\sin^2 x - \cos^2 x}{\cos x \sin x}}{\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}} \\ &= \frac{\sin^2 x - \cos^2 x}{\cos x \sin x} \cdot \frac{\cos x \cdot \sin x}{\cos x \sin x} \\ &= \sin^2 x - \cos^2 x \end{aligned}$$

$$= \sin^2 x - \cos^2 x$$