AP Calculus AB

Trigonometry Review

Ex 1) Find the 6 trig ratios of $\frac{7\pi}{4}$:

$\sin\frac{7\pi}{4} = -\frac{\sqrt{2}}{2}$ $\csc\frac{7\pi}{4} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$

$\cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2}$ $\sec\frac{7\pi}{4} = \frac{2}{\sqrt{2}} = \sqrt{2}$

$\tan\frac{7\pi}{4} = -1$ $\cot\frac{7\pi}{4} = -1$

Ex 2) Find each of the following:

a) $\sin\left(-\frac{7\pi}{6}\right)$ **b)** $\tan\frac{5\pi}{3}$ **c)** $\sec\left(-\frac{3\pi}{4}\right)$

$\frac{1}{2}$ $\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$ $-\frac{2}{\sqrt{2}} = -\sqrt{2}$

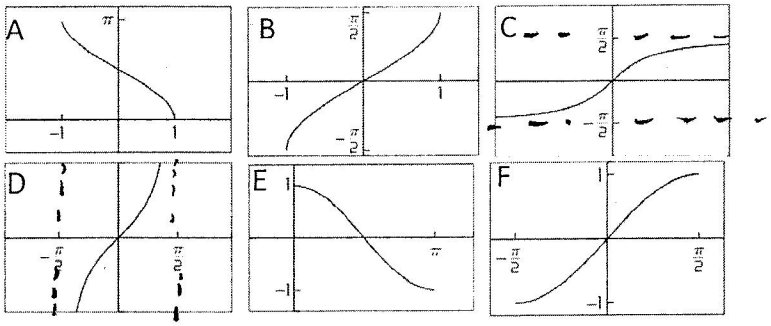
Ex 3) Find each of the following, if it exists:

a) $\sin(-270°)$ **b)** $\tan 3\pi$ **c)** $\sec(-\pi/2)$ **d)** $\tan(3\pi/2)$

1 $\frac{0}{-1} = 0$ $\frac{1}{0}$ undefined $\frac{-1}{0}$ undefined

**********Which is Which?**********

1) $y = \sin x$ F
2) $y = \cos x$ E
3) $y = \tan x$ D
4) $y = \arcsin x$ B
5) $y = \arccos x$ A
6) $y = \arctan x$ C



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Ex 4) Find the exact value of each expression without a calculator:

a) $\sin^{-1}\left(\frac{1}{2}\right)$ $\frac{\pi}{6}$

b) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ $-\frac{\pi}{3}$

c) $\sin^{-1}\left(\frac{\pi}{2}\right)$ $\frac{3.14}{2} = 1.57$ not possible

d) $\sin^{-1}\left(\sin\left(\frac{\pi}{9}\right)\right)$ pos, 1st quad $\frac{\pi}{9}$

e) $\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right)$ $\sin^{-1}\left(\frac{1}{2}\right)$ $\frac{\pi}{6}$

f) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ $\frac{3\pi}{4}$

g) $\tan^{-1}\left(\sqrt{3}\right)$ $\frac{\pi}{3}$

h) $\cos^{-1}(\cos(-1.1))$ pos, 4th quad 1.1 radians

Ex 5) Simplify each expression below using trig identities:

a) $\sin^3 x + \sin x \cos^2 x$

$\sin x(\sin^2 x + \cos^2 x)$

$\sin x \cdot (1)$

$\boxed{\sin x}$

b) $\dfrac{(\sec x + 1)(\sec x - 1)}{\sin^2 x}$

$\dfrac{\sec^2 x - \cancel{\sec x} + \cancel{\sec x} - 1}{\sin^2 x}$

$\dfrac{\tan^2 x}{\sin^2 x} = \dfrac{\frac{\sin^2 x}{\cos^2 x}}{\sin^2 x} = \dfrac{\cancel{\sin^2 x}}{\cos^2 x} \cdot \dfrac{1}{\cancel{\sin^2 x}} = \boxed{\sec^2 x}$

c) $\dfrac{\cos x}{(1-\sin x)} - \dfrac{\sin x}{\cos x}$ $\quad \cos x \cdot (1-\sin x)$, $\cos x\,(1-\sin x)$

$\dfrac{\cos^2 x - \sin x + \sin^2 x}{(1-\sin x)(\cos x)}$

$\dfrac{\cancel{1-\sin x}}{(\cancel{1-\sin x})(\cos x)} = \dfrac{1}{\cos x} = \boxed{\sec x}$

Ex 6) Find all values of x in the interval $[0, 2\pi)$ that solve each of the following equations:

$0 \le x < 2\pi$

$0 \le 3x < 6\pi$

a) $2\sin(3x) = 1$

$\sin(3x) = \frac{1}{2}$

$3x = \sin^{-1}\left(\frac{1}{2}\right)$

$3x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}$

$x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$

b) $2\sin^2 x + \sin x = 1$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0 \qquad \sin x + 1 = 0$

$\sin x = \frac{1}{2} \qquad \sin x = -1$

$x = \sin^{-1}\left(\frac{1}{2}\right) \qquad x = \sin^{-1}(-1)$

$\pi/6, 5\pi/6 \qquad \boxed{\frac{3\pi}{2}}$

Ex 7) Write each of the following as a single, simplified trig expression:

a) $2\sin 15° \cos 15°$ $= \sin(2 \cdot 15°) = \sin 30° = \frac{1}{2}$

b) $1 - 2\sin^2 22.5°$ $= \cos(2 \cdot 22.5°) = \cos 45° = \frac{\sqrt{2}}{2}$

c) $\cos 47° \cos 17° - \sin 47° \sin 17°$ $= \cos(47° + 17°) = \cos 64°$

d) $\dfrac{\tan 52.5° - \tan 7.5°}{1 + \tan 52.5° \tan 7.5°}$ $= \tan(52.5° - 7.5°) = \tan 45° = 1$