Notes -- Types of Probabilities

probability describes the chance that an uncertain event will occur.

Experimental probability of an event is an "estimate" that the event will happen based on how often the event occurs after collecting data or running an experiment (in a large number of trials). It is based specifically on direct observations or experiences.

Rule for calculating empirical probability
To compute the empirical probability, repeat a random trial a large number of times, then

If A stands for an event then, $P(\mathrm{A}) = \dfrac{\text{number of times event A occured}}{\text{number of trials}}$

Example 1 A survey was conducted to determine students' favorite breeds of dogs. Each student chose only one breed.

Dog	Collie	Spaniel	Lab	Boxer	PitBull	Other
#	10	15	35	8	5	12

What is the probability that a student's favorite dog breed is Lab?

$$p(\text{Lab}) = \frac{35}{85} = \frac{7}{17} = .412$$

Theoretical probability of an event is the number of ways that the event can occur, divided by the total number of outcomes. It is finding the probability of events that come from a sample space of known equally likely outcomes. Theoretical probability gives you an estimate of how often an event will occur.

Rule for calculating theoretical probability
When all outcomes are *equally* likely, then the theoretical probability of an event can be calculated by the following formula:

If A stands for an event then, $P(\mathrm{A}) = \dfrac{\text{number of outcomes in A}}{\text{number of all possible outcomes}}$

Example 2 Find the probability of rolling a six on a fair die.

$$p(6) = \frac{1}{6} = .167$$

Example 3 Find the probability of tossing a fair die and getting an odd number.

$$p(\text{odd}) = \frac{3}{6} = \frac{1}{2} = .5$$

Example 4 Suppose you flip a coin three times. There are 8 possible outcomes. The sample space consists of the eight possible outcomes listed below:

HHH HHT HTH HTT THH THT TTH TTT

A. Calculate the probability of the event: "Exactly 2 heads come up".

HHH HHT HTH HTT THH THT TTH TTT

$p(2 \text{ heads}) = \frac{3}{8} = .375$

B. Calculate probability of the event: "Exactly 3 tails come up".

HHH HHT HTH HTT THH THT TTH TTT

$p(3 \text{ tails}) = \frac{1}{8} = .125$

C. Calculate the probability of the event: " A tail comes up on the first flip"

HHH HHT HTH HTT THH THT TTH TTT

$p(\text{tail } 1^{st}) = \frac{4}{8} = \frac{1}{2} = .5$

theoretical

Example 5 (Comparing Empirical and Theoretical Probabilities) Karen and Jason roll two dice 50 times and record their results in the accompanying chart below.

A. What is their empirical probability of rolling a 7? $p(7) = \frac{13}{50} = .26$

B. What is the theoretical probability of rolling a 7? Use the table below. $p(7) = \frac{6}{36} = \frac{1}{6} = .167$

C. How do the empirical and theoretical probabilities compare?

close but not the same

more trials $\Rightarrow$ prob. closer to expected

Sum of the rolls of two dice
3, 5, 5, 4, 6, 7, 7, 5, 9, 10, 12, 9, 6, 5, 7, 8, 7, 4, 11, 6, 8, 8, 10, 6, 7, 4, 4, 5, 7, 9, 9, 7, 8, 11, 6, 5, 4, 7, 7, 4, 3, 6, 7, 7, 7, 8, 6, 7, 8, 9

Die 2 \ Die 1	1	2	3	4	5	6
6	7	8	9	10	11	12
5	6	7	8	9	10	11
4	5	6	7	8	9	10
3	4	5	6	7	8	9
2	3	4	5	6	7	8
1	2	3	4	5	6	7

Example 6 If a car factory checks 360 cars and 8 of them have defects, how many will have defects out of 1260?

$p(\text{defective}) = \frac{8}{360} = \frac{1}{45}$ $\quad 1260\left(\frac{1}{45}\right) = \boxed{28}$

Example 7 If a car factory checks 320 cars and 12 of them have defects, how many out of 560 will NOT have defects?

$p(\text{not defects}) = \frac{308}{320} = \frac{77}{80}$ $\quad 560\left(\frac{77}{80}\right) = \boxed{539}$

Notes -- Odds vs Probability

The **probability** of an event is a ratio that tells how likely it is that an event will take place. The numerator is the number of favorable outcomes and the denominator is the number of possible outcomes. Probability considers the context of the entire event space.

The **odds** of an event occurring is the ratio of the number of ways the event can occur (successes) to the number of ways the event cannot occur (failures)

$$P(A) = \frac{Number\ of\ Event\ A}{Total\ Number\ of\ Events} \qquad O = \frac{\text{proportion of successes}}{\text{proportion of failures}} = \frac{p}{1-p}$$

Odds = successes : failures

Example 1 At Acme Pizza on a Friday night there is a group of 25 people, 10 of which are teens. Suppose you want to pick one person from the group to receive a special dessert.

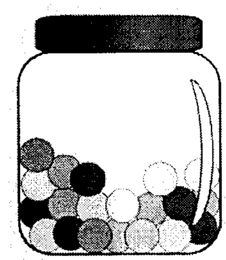
a. What is the probability of selecting a teen?

$$p(\text{teen}) = \frac{10}{25} = \frac{2}{5} = .4$$

b. What are the odds that the person is a teen?

$$\frac{10}{15} = \frac{2}{3} \quad 2:3$$

Example 2 One marble will be pulled out of a bag containing 12 marbles. There are 6 blue, 3 red, 2 yellow, and 1 green for a total of 12 marbles in the bag.



A. Find the probability of pulling a red marble.

$$p(\text{red}) = \frac{3}{12} = \frac{1}{4} = .25$$

B. Find the 'odds in favor' of RED.

$$\frac{3}{9} = \frac{1}{3} \quad 1:3$$

C. Find the 'odds against' RED. Hint: flip odds in favor upside down, and this describes the odds of the event not occurring.

$$\frac{9}{3} = \frac{3}{1} \quad 3:1$$

Example 3 Eleven poker chips are numbered consecutively 1 through 10, with two of them labeled with a 6. The chips are placed in a jar. A chip is drawn at random.

A. Find the probability of drawing a 6. $p(6) = \frac{2}{11} = .182$

B. Find the odds of drawing a 6. $\frac{2}{9}$ $2:9$

C. Find the odds of not drawing a 6. $\frac{9}{2}$ $9:2$

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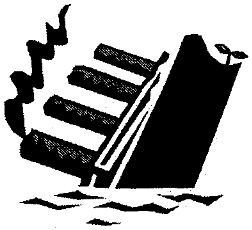
Example 4 Suppose that 38% of passengers survived the sinking of the Titanic. What was the overall odds of survival? Interpret your answer



$$\frac{.38}{.62} = \frac{.613}{1}$$

For every 1 person who died, there was an avg. of .61 survivor

Example 5 Suppose 19% of men survived the sinking of the Titanic, while 73% of women survived. Calculate the odds of survival for each group. Find the ratio of these odds and interpret your answer

women: $\frac{.73}{.27} = 2.704$

men: $\frac{.19}{.81} = .235$

$$\frac{2.704}{.235} = \frac{11.506}{1}$$

Women were 11.5 times more likely to survive than men

Example 6 The number of males and females enrolled in Blue Dolphin High School are listed per class in the table below.

Blue Dolphin High School		
Grade	Male	Female
9	120	150
10	100	100
11	130	110
12	150	175
	500	535

A. If a student is chosen at random, what is the probability that the student is a female?

$$p(\text{female}) = \frac{535}{1035} = \frac{107}{207} = .517$$

B. If a student is chosen at random, what is the probability that the student is a male in Grade 11?

$$p(11^{th}\text{ grade male}) = \frac{130}{1035} = \frac{26}{207} = .126$$

C. If one student is chosen to represent the student body, what are the odds in favor of selecting a female?

$$\frac{535}{500} = \frac{107}{100}$$

D. If one student is chosen from Grade 12, which is more likely, selecting a male or selecting a female?