

NOTE -- not all answers are verified  
(only those that are worked out)

Name: \_\_\_\_\_

**Review for Test #3 (A.3, 2.7-2.9)**

- (A.3) Simplifying, Multiplying, Dividing, Adding, Subtracting Rational Expressions & Complex Fractions
- (2.7) Graphing Rational Functions
- (2.8) Solving Rational Equations
- (2.9) Solving Polynomial & Rational Inequalities

**(A.3) Operations With Rational Expressions-----KEY POINTS TO REMEMBER**

- \* ALWAYS factor 1<sup>ST</sup>!!!!!!
- \* You DO NOT need LCD when multiplying and dividing
- \* Remember how you work with regular fractions to add them, follow the SAME process
- \* When adding/subtracting DO NOT cancel the factors YOU multiplied in to make the LCD before you add/subtract

Perform each of the following operations, write your answer in the SIMPLIEST form possible, & state the restrictions.

1)  $\frac{x^2 + 6x + 5}{x^2 + 3x - 10}$

$\frac{(x+5)(x+1)}{(x-2)(x+5)} = \frac{x+1}{x-2}$   
 $x \neq 2, -5$

2)  $\frac{6x^2 - 7x - 3}{8x^2 - 2x - 15}$

$\frac{(3x+1)(2x-3)}{(4x+5)(2x-3)}$   
 $\frac{3x+1}{4x+5}$   $x \neq -\frac{5}{4}, \frac{3}{2}$

3)  $\frac{20x^3 + 30x^2 + 45x}{40x^4 - 135x} = \frac{1}{2x-3}$   
 $x \neq 0, \frac{3}{2}$

4)  $\frac{x^2 + 16x + 55}{x^2 - 8x - 65} \cdot \frac{x^3 - 11x^2 - 26x}{x^2 + 13x + 22}$

$\frac{(x+5)(x+11)}{(x-13)(x+5)} \cdot \frac{x(x-11)(x+2)}{(x+2)(x+11)}$

$\frac{x}{x}$   $x \neq 13, -5, -2, -11$

5)  $\frac{x^4 - 1}{x^3 - 3x^2 + x - 3} \cdot \frac{4x^2 - 7x - 15}{(x-1)(x+1)(4x+5)}$

$x \neq 3$

6)  $\frac{x^3 + 64}{x^2 - 16} \div x^2 - 4x + 16$

$\frac{1}{x-4}$ ,  $x \neq -4, 4$

7)  $\frac{5x^2}{2x^2 + 5x - 33} \div \frac{5x(x^2 - 4)}{5x^3 - 20x}$

$\frac{5x^2}{(2x+11)(x-3)} \cdot \frac{(2x+11)(x+2)}{5x(x+2)(x-2)}$

$\frac{x}{(x-3)(x-2)}$   $x \neq -\frac{11}{2}, 3, -2, 2, 0$

8)  $\frac{x-1}{x^2 - 17x + 72} + \frac{-x}{x^2 - 3x - 54}$

$\frac{13x-6}{(x-9)(x-8)(x+6)} \times \frac{(3x-10)}{(3x-10)(x+1)(x-1)} - \frac{5(x-1)}{(3x-10)(x+1)(x-1)}$

LCD =  $(x+1)(x-1)(3x-10)$

$\frac{3x^2 - 10x - 5x + 5}{(3x-10)(x+1)(x-1)}$

$\frac{3x^2 - 15x + 5}{(3x-10)(x+1)(x-1)}$   $x \neq -1, \frac{10}{3}, 15$

$$10) \frac{\frac{x}{3} + 5}{7 + \frac{6}{x}}$$

$$\frac{x^2 + 15x}{21x + 18}$$

$$x \neq 0$$

$$11) \frac{5}{\frac{1}{x-2} + \frac{2}{x+1}}$$

LCD = (x-2)(x+1)

$$12) \frac{(x+4)^2}{\frac{(x+5)^2}{x+4}} = \frac{x+4}{\frac{1}{x+5}}$$

$$x \neq -5, -4$$

$$13) \frac{(x+y)^3}{\frac{(x+y)^2}{x^2+2xy+y^2}} = 1$$

$$x \neq -y$$

$$\frac{1(x+1) + 2(x-2)}{(x-2)(x+1)} \rightarrow \frac{x+1+2x-4}{(x-2)(x+1)} \rightarrow \frac{3x-3}{(x-2)(x+1)} \rightarrow \frac{3(x-1)}{(x-2)(x+1)}$$

$$\frac{5}{\frac{3(x-1)}{(x-2)(x+1)}} = \frac{5}{x-2} \cdot \frac{(x-2)(x+1)}{3(x-1)} = \frac{5(x+1)}{3(x-1)}$$

$$x \neq 2, -1, 1$$

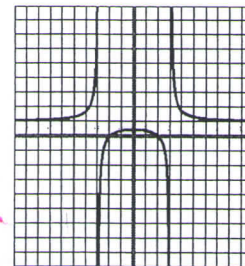
(2.7) Graphing Rational Functions-----KEY POINTS TO REMEMBER

- \* ALWAYS factor 1<sup>ST</sup>, if the expression simplifies there is a HOLE in the graph.
- \* If there is a hole in the graph, ALL FURTHER CALCULATIONS should be done using the SIMPLIFIED expression
- \* Real zeros of the numerator (using the SIMPLIFIED version) are x-intercepts of the function
- \* Real zeros of the denominator (using the SIMPLIFIED version) are the locations of the VERTICAL ASYMPTOTES
- \* Look at your notes for the THREE situations to determine the end behavior asymptotes of the function
- \* Look at your notes or the textbook to determine what the following notation means  $\lim_{x \rightarrow -1^+}$
- \* Asymptotes are written as EQUATIONS (look in your notes/textbook to clarify)

14) Use the graph to the right to fill in the blanks:

$$\lim_{x \rightarrow -\infty} = 1 \quad \lim_{x \rightarrow -3^-} = \infty \quad \lim_{x \rightarrow -3^+} = -\infty$$

$$\lim_{x \rightarrow 3^-} = -\infty \quad \lim_{x \rightarrow 3^+} = \infty \quad \lim_{x \rightarrow \infty} = 1$$



15) Determine the holes, intercepts, asymptotes, and then sketch each of the following:

a)  $f(x) = \frac{x^2 - 4}{x^2 - 9} = \frac{(x+2)(x-2)}{(x+3)(x-3)}$

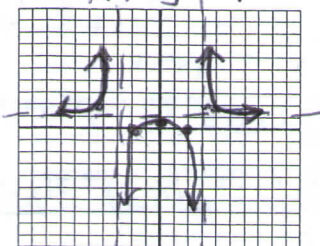
Hole(s): (none)

x-int: (-2, 0) (2, 0)

y-int: (0, 4/9)

Eqs of ALL

Asymptotes: V.A.  $x = -3, 3$   
H.A.  $y = 1$



x	y
-4	$\frac{16-4}{16-9} = \frac{12}{7}$
4	$\frac{16-4}{16-9} = \frac{12}{7}$

b)  $f(x) = \frac{3x^2 - x - 4}{9x^3 + 9x^2 - 16x - 16}$

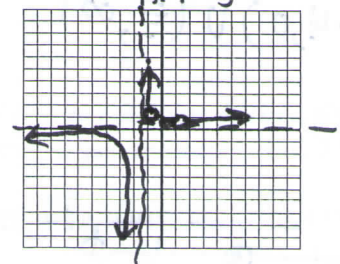
Hole(s): (-1, 1) (4/3, 1/8)

x-int: (none)

y-int: (0, 1/4)

Eqs of ALL

Asymptotes: VA  $x = -4/3$   
HA  $y = 0$



(2.8) Solving Rational Equations-----KEY POINTS TO REMEMBER

- \* Factor 1<sup>st</sup> & Simplify if you can
- \* Find the LCD & multiply EVERY SINGLE term in the equation by that LCD to clear ALL fractions.
- \* CHECK EVERY ANSWER!!!!

16)  $\frac{x-2}{x+4} + \frac{x+1}{x+6} = \frac{11x+32}{x^2+10x+24}$

$\frac{x-2}{x+4} + \frac{x+1}{x+6} = \frac{11x+32}{(x+6)(x+4)}$

LCD = (x+4)(x+6)

$\frac{(x-2)(x+4)(x+6)}{x+4} + \frac{(x+1)(x+4)(x+6)}{x+6} = \frac{(11x+32)(x+4)(x+6)}{(x+6)(x+4)}$

18)  $\frac{3x}{x+5} + \frac{1}{x-2} = \frac{7}{x^2+3x-10}$

$x = -1/3$

17)  $\frac{y+3}{y+2} = 1 - \frac{y+1}{y+2}$  no soln.

$x^2+6x-2x-12+x^2+4x+x+4 = 11x+32$

$2x^2+9x-8 = 11x+32$

$2x^2-2x-40 = 0$

$2(x^2-x-20) = 0$

$2(x-5)(x+4) = 0$

$x = 5, x = -4$

19)  $\frac{2}{x-3} - \frac{3}{4-x} = \frac{2x-2}{x^2-x-12}$

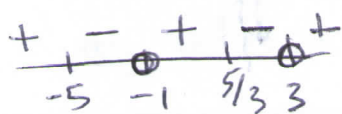
$x = -1 \pm 2\sqrt{5}$

(2.9) Solving Rational & Polynomial Inequalities-----KEY POINTS TO REMEMBER

- \* Factor 1<sup>st</sup> & Simplify if you can
- \* If there are rational expressions on BOTH sides you MUST move them to ONE side
- \* You CANNOT EVER multiply an INEQUALITY by the LCD on both sides
- \* When using a SIGN CHART SHOW ALL WORK!!!
- \* If using a sketch of the polynomial to solve, then you MUST CLEARLY show the LABELED SKETCH

20)  $\frac{3}{4} + \frac{x}{2} > \frac{5}{x}$

$(-4, 0) \cup (\frac{5}{2}, \infty)$



$(-\infty, -5] \cup (-1, \frac{5}{3}] \cup (3, \infty)$

21)  $\frac{6}{x+3} > x+8$   
 $(-\infty, -9) \cup (-3, -2)$

$\frac{3x^2+10x-25}{(x-3)(x+1)} \geq 0$

$\frac{(3x-5)(x+5)}{(x-3)(x+1)} \geq 0$

zero:  $\frac{5}{3}, -5$

undef:  $3, -1$

pos & zero

22)  $\frac{8}{x-3} + \frac{8}{x+1} \geq -3$

$\frac{8}{x-3} + \frac{8}{x+1} + 3 \geq 0$

LCD = (x-3)(x+1)

$\frac{8(x+1)+8(x-3)+3(x-3)(x+1)}{(x-3)(x+1)} \geq 0$

$\frac{8x+8+8x-24+3x^2+3x-9x-9}{(x-3)(x+1)} \geq 0$

$(x-3)(x+1)$

$$23) \frac{10}{x-4} + x \geq \frac{3x-2}{x-4}$$

$$[3, 4) \cup (4, \infty)$$

$$24) (2x+4)(x-3) > 0$$

$$(-\infty, -2) \cup (3, \infty)$$

$$25) \frac{(x-1)^2}{(x+1)(x+2)} > 0$$

$$(-\infty, -2) \cup (-1, 1) \cup (1, \infty)$$

$$26) \frac{x^2-4}{x^2+4} \geq 0$$

$$(-\infty, -2] \cup [2, \infty)$$

$$27) \frac{3+x}{3-x} \geq 1$$

$$[0, 3)$$

$$28) x^3 + 9x^2 + 20x + 12 < 0$$

constant: 1, 2, 3, 4, 6, 12

l.c.: 1

possible rational zeros:

$\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

$$\begin{array}{r} -1 \mid 1 \quad 9 \quad 20 \quad 12 \\ \quad \quad -1 \quad -8 \quad -12 \\ \hline 1 \quad 8 \quad 12 \quad 0 \end{array}$$

$x^2 + 8x + 12$   
 $(x+6)(x+2)$

$(x+6)(x+2)(x+1) < 0$

$\begin{array}{cccc} - & + & - & + \\ -6 & -2 & -1 & \end{array}$

**STUDY CHECK LIST:**

- Re-read your notes ALOUD
- Finish & check your answers to this review packet
- Look over your previous homework assignments & be sure you are **ABLE** to do **EVERY SINGLE** problem
- If you still have questions on a particular topic/type of question, **GOOGLE** it, look for a tutorial online
- DO NOT** expect to ask further clarifying questions @ the beginning of class before the test

**MORE SUGGESTIONS:**

- Form a study group
- Re-work the example questions from your notes, paying attention to what you are doing & WHY, then check
- **TEACH** someone else, the topics from this section (a parent, sibling, friend, or just pretend to teach & explain it to someone else)

\*\*\*\*\*THIS IS BACKED BY RESEARCH TO BE THE MOST EFFECTIVE WAY TO LEARN AND RETAIN INFORMATION\*\*\*\*\*

Name \_\_\_\_\_

**Rational Expression Worksheet #8:  
Simplify/Multiply/Divide**

Simplify (remember to factor when necessary).

$$1. \frac{18x^6}{27x^4} = \frac{2x^2}{3}$$

$$2. \frac{x^2+6x+8}{3x+12} = \frac{(x+4)(x+2)}{3(x+4)} = \frac{x+2}{3}$$

$$3. \frac{x^2-7x+12}{x^2+2x-15} = \frac{(x-3)(x-4)}{(x+5)(x-3)} = \frac{x-4}{x+5}$$

Multiply or divide (remember to factor when necessary).

$$4. \frac{x+3}{x^2-4x+4} \cdot \frac{x^2-x-2}{x^2+4x+3} = \frac{x+3}{(x-2)(x-2)} \cdot \frac{(x-2)(x+1)}{(x+3)(x+1)} = \frac{1}{x-2}$$

$$5. \frac{x^2-x-12}{3x+9} \div \frac{x^2+x-20}{x+5} = \frac{(x-4)(x+3)}{3(x+3)} \cdot \frac{x+5}{(x+5)(x-4)} = \frac{1}{3}$$

$$6. \frac{15x^2}{45x^3} \div \frac{5x^6}{9x^4}$$

same as front

$$\frac{3}{4x^3}$$

$$7. \frac{6}{x^2-9x+20} \cdot \frac{5x-25}{3x-6} = \frac{6}{(x-4)(x-5)} \cdot \frac{5(x-5)}{3(x-2)} = \frac{10}{3(x-4)(x-2)}$$

$$8. \frac{6x-12}{4x^2} \cdot \frac{3x^3}{2x-4} = \frac{6(x-2)}{4x^2} \cdot \frac{3x^3}{2(x-2)} = \frac{18x^3}{8x^2} = \frac{9x}{4}$$

$$9. \frac{3x-21}{x^2-3x-28} \cdot \frac{5x+20}{2x+8} = \frac{3(x-7)}{(x-7)(x+4)} \cdot \frac{5(x+4)}{2(x+4)} = \frac{15}{2(x+4)}$$

$$10. \frac{x^2-5x-6}{2x+6} \div \frac{x^2-3x-4}{4x+12} = \frac{(x-6)(x+1)}{2(x+3)} \cdot \frac{4(x+3)}{(x-4)(x+1)} = \frac{x-6}{2(x-4)}$$

$$11. \frac{25xy^3}{35x^4y^2} \cdot \frac{14xy}{10x^2y^3} = \frac{350x^2y^4}{350x^6y^5} = \frac{1}{x^4y}$$

$$12. \frac{4x}{x+1} \cdot \frac{x^2-6x-7}{x^2-7x} = \frac{4x}{x+1} \cdot \frac{(x-7)(x+1)}{x(x-7)} = \frac{4}{x}$$

$$13. \frac{6x-30}{x^2-7x+10} \cdot \frac{7x-14}{6x} = \frac{6(x-5)}{(x-5)(x-2)} \cdot \frac{7(x-2)}{6x} = \frac{7}{x}$$

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Name

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Date:

Algebra 2

**Rational Expression Worksheet:  
Adding/Subtracting Common Denominator**

Add or subtract these rational expressions.

1.  $\frac{9}{15x} + \frac{2}{15x} = \frac{11}{15x}$

2.  $\frac{5x}{7} - \frac{2x}{7} = \frac{3x}{7}$

3.  $\frac{4x}{2x+3} + \frac{7}{2x+3} = \frac{4x+7}{2x+3}$

4.  $\frac{2}{5x+9} + \frac{x}{5x+9} = \frac{2+x}{5x+9}$

5.  $\frac{5}{8a} - \frac{2}{8a} = \frac{3}{8a}$

6.  $\frac{7}{x-5} - \frac{4}{x-5} = \frac{3}{x-5}$

7.  $\frac{y}{y^2-9} + \frac{5}{y^2-9} = \frac{y+5}{y^2-9}$

8.  $\frac{8}{2x^2} + \frac{3}{2x^2} = \frac{11}{2x^2}$

9.  $\frac{2}{x+1} + \frac{1}{x+1} = \frac{3}{x+1}$

10.  $\frac{x-1}{3x+4} + \frac{2x+9}{3x+4} = \frac{3x-8}{3x+4}$

11.  $\frac{5x}{3x^2} - \frac{4}{3x^2} = \frac{5x-4}{3x^2}$

12.  $\frac{7x+4}{x^2+3x+2} - \frac{3x-2}{x^2+3x+2} = \frac{4x+6}{x^2+3x+2}$

Name \_\_\_\_\_

### Rational Expression Worksheet #13: Solving Equations

Solve each equation for  $x$ . SHOW WORK!

$$1.) \frac{15}{x-6} + \frac{7x}{x-6} = \frac{-6}{x-6}$$

$$x = -3$$

$$2.) \frac{11x}{4x+9} - \frac{14}{4x+9} = \frac{41}{4x+9}$$

$$x = 5$$

$$3.) \frac{3x}{7x} + \frac{1}{7} = \frac{4}{x}$$

$$x = 7$$

$$4.) \frac{2x}{3x} - \frac{5}{6} = \frac{5}{2x}$$

$$x = -15$$

$$5.) \frac{1}{7(x-3)} + \frac{4}{7} = \frac{3}{(x-3)}$$

$$x = 8$$

$$6.) \frac{2}{5} - \frac{7}{(x+6)} = \frac{9}{5(x+6)}$$

$$x = 16$$