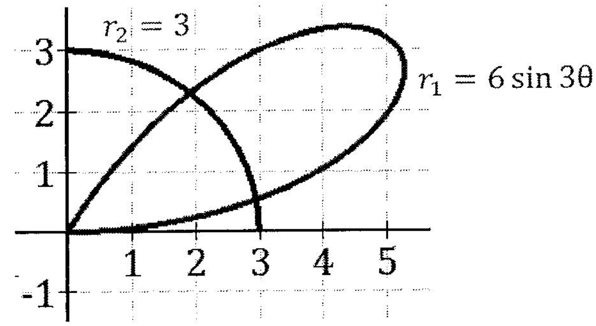
Derivatives and Equations in Polar Coordinates

1. The graphs of the polar curves $r_1 = 6\sin 3\theta$ and $r_2 = 3$ are shown to the right.
(You may use your calculator for all sections of this problem.)
   a) Find the coordinates of the points of intersection of both curves for $0 \leq \theta < \frac{\pi}{2}$. Write your answers using polar coordinates.
   b) Write the coordinates of the points of intersection using rectangular coordinates.



   c) Find $\left.\frac{dr_1}{d\theta}\right|_{\theta=\frac{\pi}{4}}$. Interpret the meaning of your answer in the context of the problem.
   d) For $0 \leq \theta < \frac{\pi}{2}$, there are two points on r_1 with x-coordinate equal to 4. Find the subject points. Express your answer using polar coordinates.
   e) Write in terms of θ an expression for $\frac{dy}{dx}$, the slope of the tangent line to the graph of r_1.
   f) Write in terms of x and y an equation for the line tangent to the graph of the curve r_1 at the point where $\theta = \frac{\pi}{4}$.

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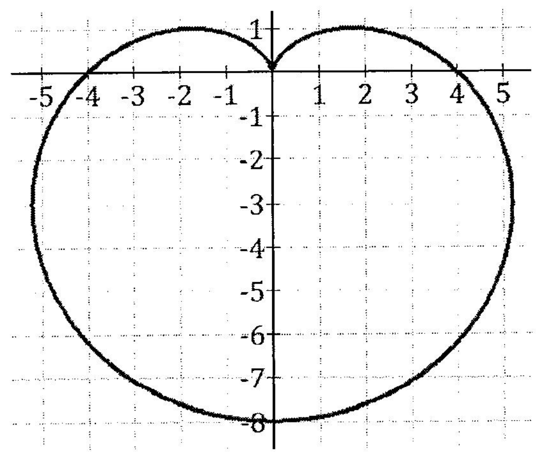
2. The graph of the polar curve $r = 4 - 4\sin\theta$ is shown to the right.
(You may use your calculator for all sections of this problem.)
   a) For $0 \leq \theta < 2\pi$, there are two points on r with y-coordinate equal to -4. Find the subject points. Express your answers using polar coordinates.
   b) Write an expression for the x-coordinate of each point on the graph of $r = 4 - 4\sin\theta$. Express your answer in terms of θ.
   c) A particle moves along the polar curve $r = 4 - 4\sin\theta$ so that at time t seconds, $\theta = t^2$. Find the time t in the time interval $1 \leq t \leq 2$ for which the x-coordinate of the particle's position is -1.



   d) Find $\left.\frac{dr}{dt}\right|_{t=2}$. Interpret the meaning of your answer in the context of the problem.
   e) Find $\left.\frac{dx}{dt}\right|_{t=2}$. Interpret the meaning of your answer in the context of the problem.

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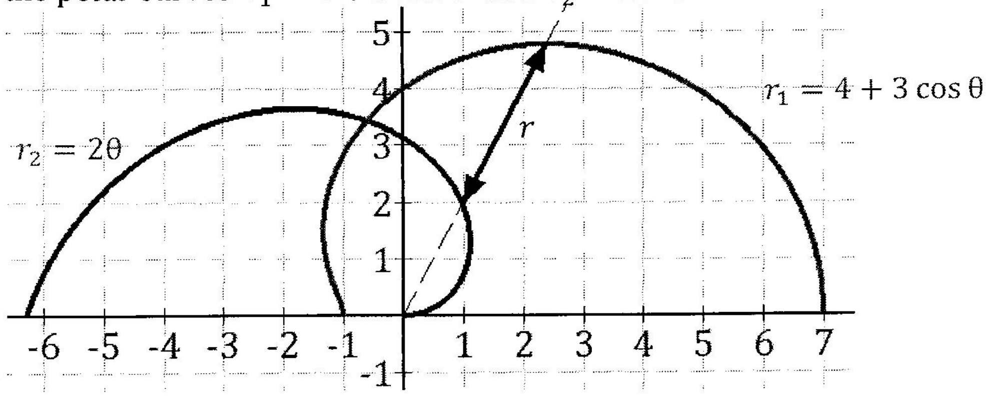
3. The graphs of the polar curves $r_1 = 4 + 3\cos\theta$ and $r_2 = 2\theta$ are shown below.



(Do NOT use your calculator for this problem unless indicated!)

a) Find the coordinates of the points of intersection of both curves for $0 \le \theta < \pi$. Write your answer using polar coordinates. *(You may use your calculator for this section.)*

b) As the curves are traced, the distance between them, $r(\theta)$, changes (see drawing.) Find an expression for $r(\theta)$ the distance between both curves in the interval $0 \le \theta \le \frac{\pi}{2}$.

c) Write in terms of θ an expression for $\frac{dr}{d\theta}$. Use your answer to find $\left.\frac{dr}{d\theta}\right|_{\theta=\frac{\pi}{3}}$. Interpret the meaning of your answer in the context of the problem.

d) Write in terms of θ an expression for $\frac{dy}{dx}$, the slope of the tangent line to the graph of r_2.

e) Find the coordinates of the point where curve r_2 has a horizontal tangent line in the interval $0 < \theta < \pi$. Write your answer using rectangular coordinates. *(You may use your calculator for this section.)*

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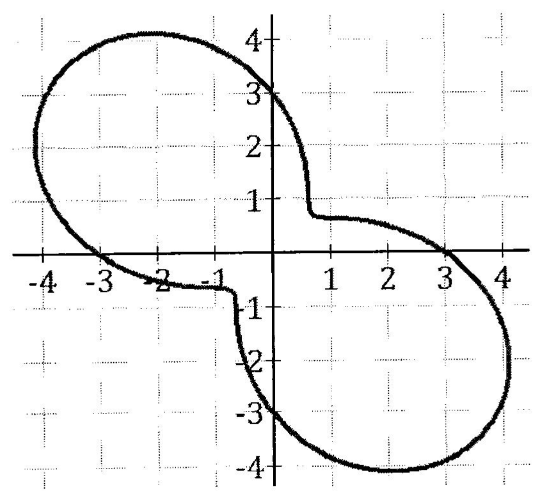
4. The graph of the polar curve $r = 3 - 2\sin(2\theta)$ for $0 \le \theta < 2\pi$ is shown to the right.
(You may use your calculator for all sections of this problem.)



a) Write in terms of θ an expression for $\frac{dy}{dx}$, the slope of the tangent line to the graph of r.

b) Find the coordinates of the point where curve r has a vertical tangent line in the interval $0 \le \theta < \pi$. Write your answer using polar coordinates.

c) Write in terms of x and y an equation for the line tangent to the graph of the curve r at the point where $\theta = \frac{\pi}{6}$.

d) A particle moves along the polar curve $r = 3 - 2\sin(2\theta)$ so that $\frac{d\theta}{dt} = 2$ for all times $t \ge 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$. Interpret the meaning of your answer in the context of the problem.

e) Assume now that for the particle whose motion was described in section (d) we have $\theta = 2t$. Find the position vector of the particle $\langle x(t), y(t)\rangle$ in terms of t. Use your calculator to find the velocity vector and the speed of the particle at $t = 1.5$.