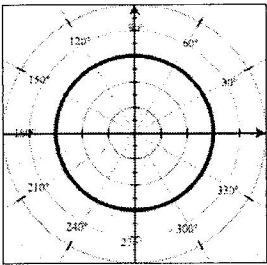


Classifying Polar Graphs

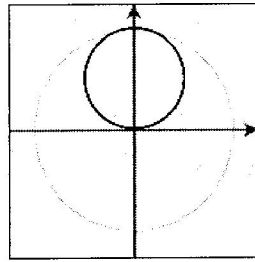
Just as was true with rectangular graphs, there are graphs in polar form that occur all the time and students should be able to recognize them by their equations.



$r = a$

Circle

center at the pole
radius = a

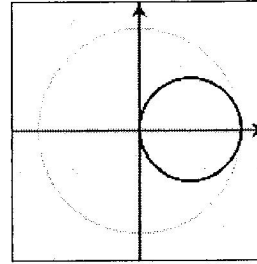


$r = a \sin \theta$

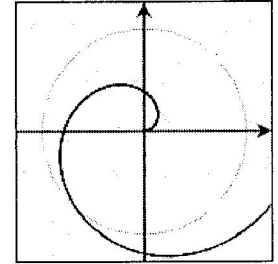
Circles not centered at the pole

a is the diameter

sine curves are symmetric to y-axis *cosine* curves are symmetric to x-axis



$r = a \cos \theta$

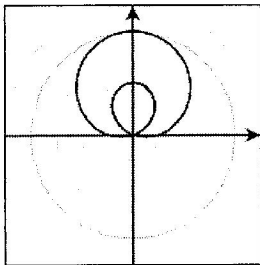


$r = a\theta$

Spiral of Archimedes

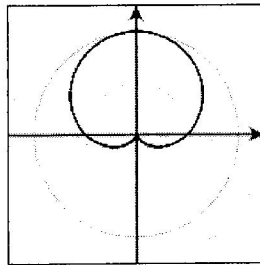
a controls the width
(must be in radian mode)

Limaçons are in the form $r = a \pm b \sin \theta$ (symmetric to y -axis) or $r = a \pm b \cos \theta$ (symmetric to x -axis)



$a < b$

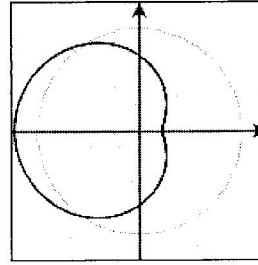
Limaçon with inner loop



$a = b$

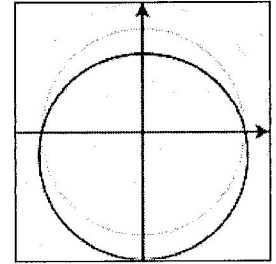
Cardioid

(heart shaped)



$b < a < 2b$

Dimpled Limaçon

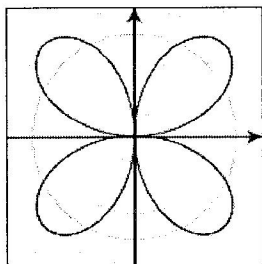


$a \geq 2b$

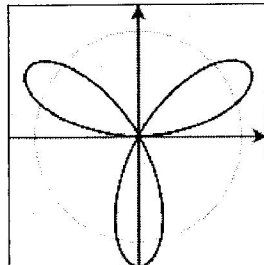
Convex Limaçon

(one side is flattened)

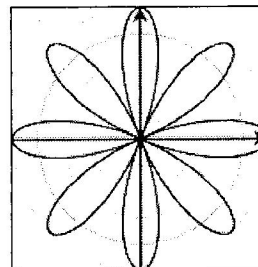
Rose curves are in the form $r = a \sin(n\theta)$ or $r = a \cos(n\theta)$. The maximum diameter of a petal is controlled by a . If n is even, the rose curve will have $2n$ petals. If n is odd, the rose curve will have n petals. Interesting patterns can be formed if n is a decimal and the curve is viewed with θ starting at 0 and going out to very large numbers.



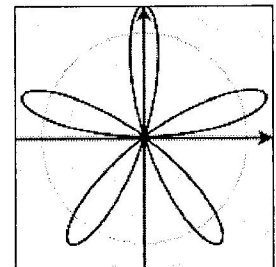
$r = a \sin(2\theta)$



$r = a \sin(3\theta)$



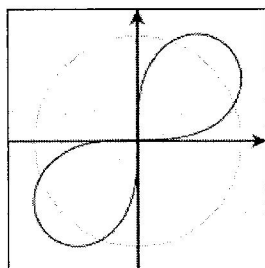
$r = a \cos(4\theta)$



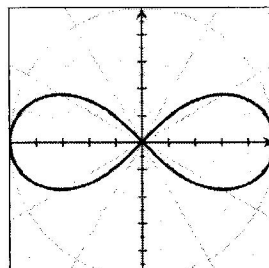
$r = a \sin(5\theta)$

Lemniscates look like infinity signs and are in the form $r^2 = a^2 \sin(2\theta)$ [symmetric to the origin] and $r^2 = a^2 \cos(2\theta)$ [symmetric to the x-axis]

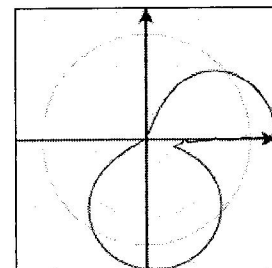
If the coefficient of θ is a number other than 1 or 2, a deformed lemniscates will result.



$r^2 = a^2 \sin(2\theta)$



$r^2 = a^2 \cos(2\theta)$



$r^2 = a^2 \cos(1.3\theta)$

Example 1 – Match the polar equations with their graphs below.

___ 1) $r = 3 - \cos\theta$

___ 2) $r = 2 - 2\sin\theta$

___ 3) $r = 5\cos(3\theta)$

___ 4) $r = 2 - 2\cos\theta$

___ 5) $r = 3 + 1.5\sin\theta$

___ 6) $r = 3.5\cos(2\theta)$

___ 7) $r = 5\sin(3\theta)$

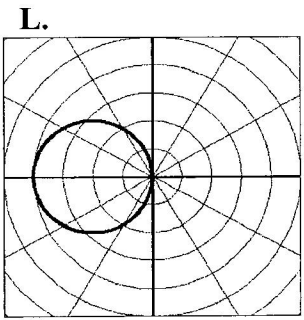
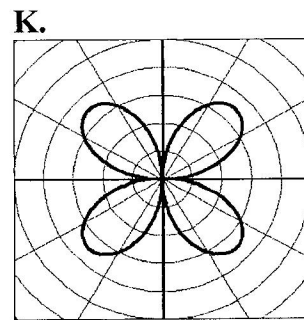
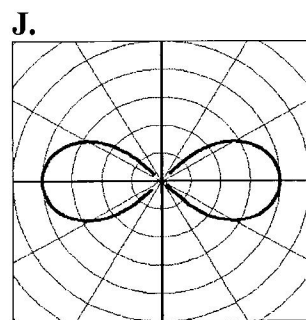
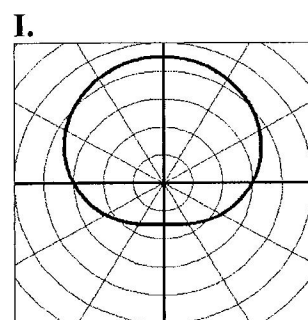
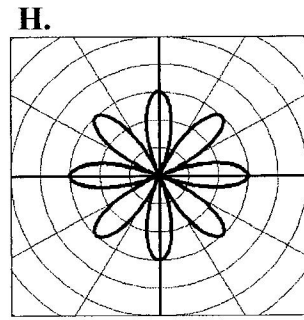
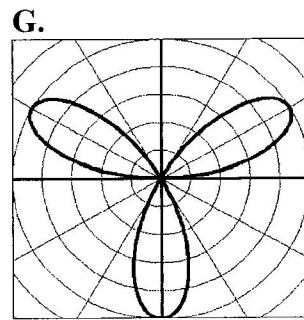
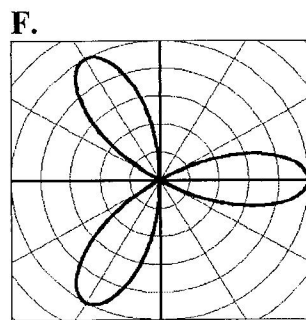
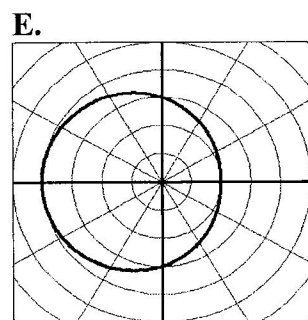
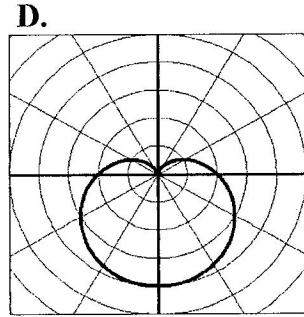
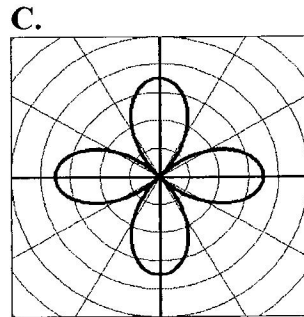
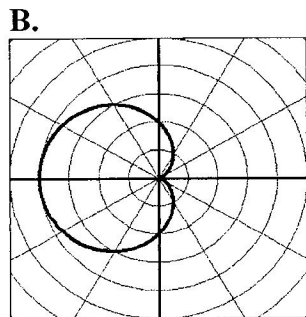
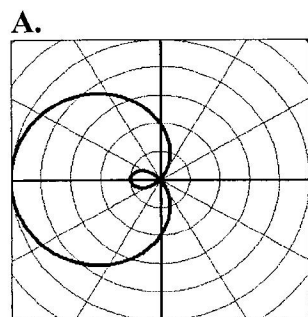
___ 8) $r^2 = -16\cos(2\theta)$

___ 9) $r = 2 - 3\cos\theta$

___ 10) $r = 3\cos(4\theta)$

___ 11) $r = -4\cos\theta$

___ 12) $r = 3.5\sin(2\theta)$



Example 2 – Match the polar equations with their graphs below.

___ 1) $r = 2.5 + 2.5\sin\theta$

___ 2) $r = 3$

___ 3) $r = 3.5\sin(3\theta)$

___ 4) $r = 4.5 \sin(2\theta)$

___ 5) $r = 4.5\cos(2\theta)$

___ 6) $r = 1.5 + 2\cos\theta$

___ 7) $r = -3\sin\theta$

___ 8) $r = 2 - \sin\theta$

___ 9) $r^2 = 16\sin(2\theta)$

___ 10) $r = 4\cos(5\theta)$

___ 11) $r = 3.5\cos(3\theta)$

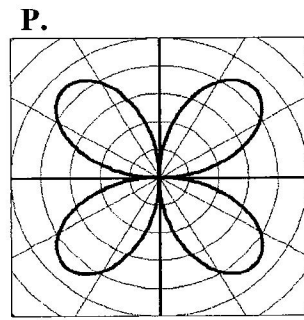
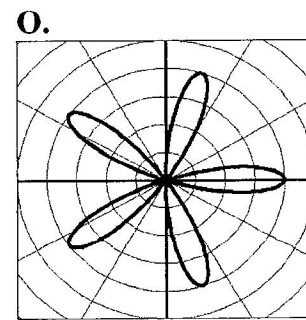
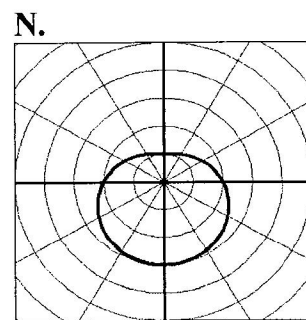
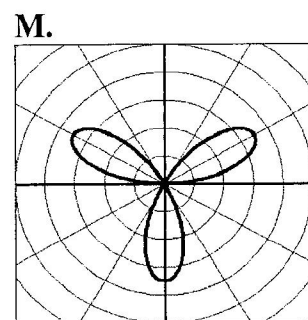
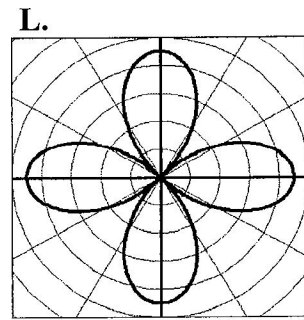
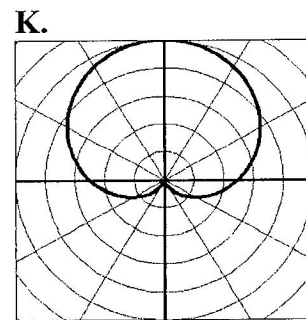
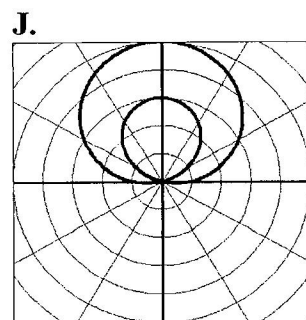
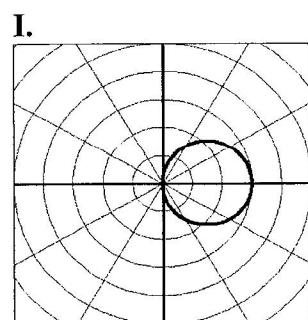
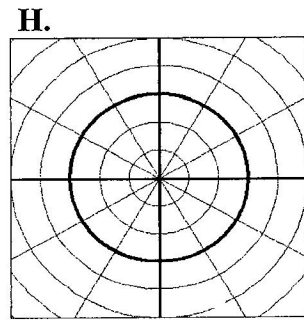
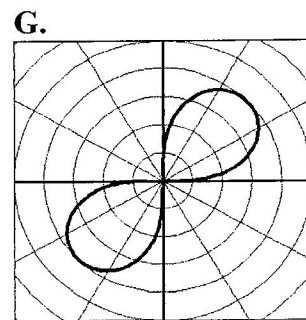
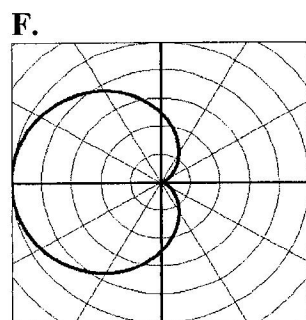
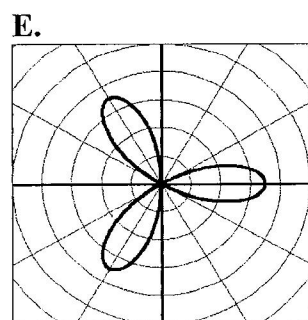
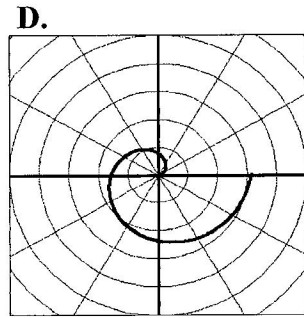
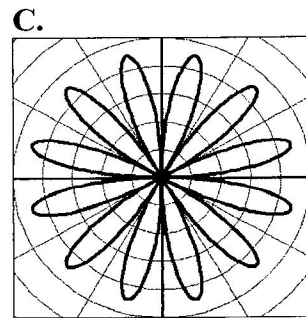
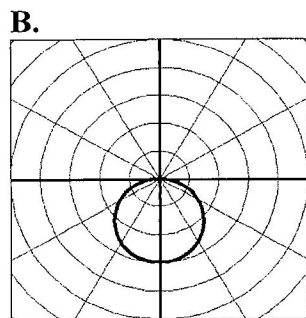
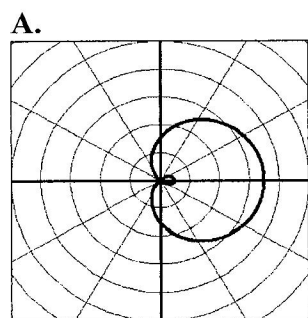
___ 12) $r = 2.5 - 2.5\cos\theta$

___ 13) $r = 3\cos\theta$

___ 14) $r = 1 + 4\sin\theta$

___ 15) $r = 4.5\sin(6\theta)$

___ 16) $r = \frac{1}{2}\theta$



Practice 4.4: Polar Graphs

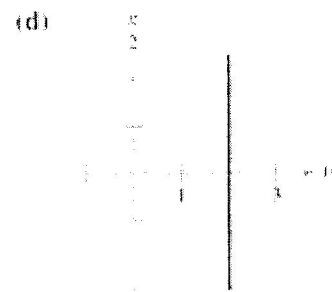
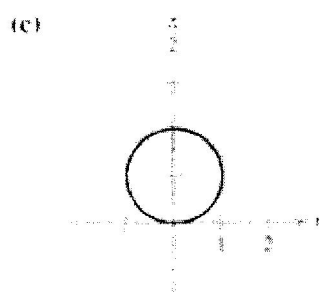
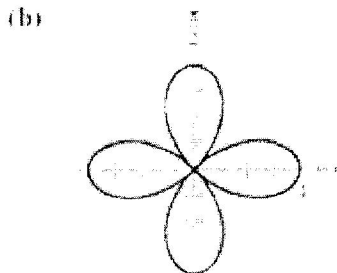
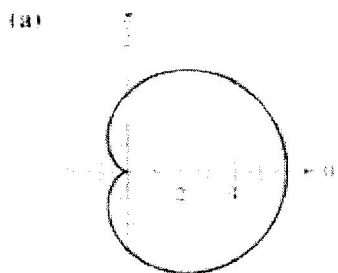
Match the graph with its polar equation.

1) $r = 2 \sin \theta$

2) $r = 4 \cos 2\theta$

3) $r = 3(1 + \cos \theta)$

4) $r = 2 \sec \theta$



5) Identify the polar graph (circle, spiral, cardioid, limacon, rose):

If the graph is a circle, name the center (in polar coordinates) and the radius.

If the graph is a limacon, name the type.

If the graph is a rose, state the number of petals.

1. $r = 4 \cos \theta$

7. $r = -2 \sin \theta$

2. $r = 5 - 2 \sin \theta$

8. $r = 6 + 6 \cos \theta$

3. $r = -7 \sin 10 \theta$

9. $r = 8 \cos 5 \theta$

4. $r = 6 \theta$

10. $r = -8$

5. $r = 4 + 7 \sin \theta$

11. $r = 8 + 6 \cos \theta$

6. $r = \frac{4}{\theta}$