Review for Unit 4 Test

1 For $0 \le t \le 13$, an object travels along an elliptical path given parametrically by $\begin{cases} x = 3\cos t \\ y = 4\sin t \end{cases}$. At the point at which $t = 13$, the object leaves the path and travels along the line tangent to the path at that point. What is the slope of the line on which the object travels?

a $-\frac{4}{3}$ b $-\frac{3}{4}$ c $-\frac{4\tan 13}{3}$ d $-\frac{4}{3\tan 13}$ e $-\frac{3}{4\tan 13}$

2 The position of a particle moving in the xy-plane is given by the parametric equations $\begin{cases} x = t^3 - 3t^2 \\ y = 2t^3 - 3t^2 - 12t \end{cases}$. For what values of t is the particle at rest?

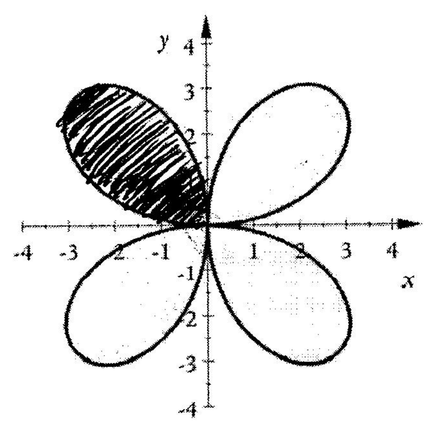
a -1 only b 0 only c 2 only d -1 and 2 only e -1, 0, and 2

3 A curve C is defined by the parametric equations $\begin{cases} x = t^2 - 4t + 1 \\ y = t^3 \end{cases}$. Which of the following is an equation of the line tangent to the graph of C at the point $(-3, 8)$?

a $x = -3$ b $x = 2$ c $y = 8$ d $y = -\frac{27}{10}(x+3)+8$ e $y = 12(x+3)+8$

4 A particle moves so that its position at time t is given by $\begin{cases} x = t^2 \\ y = \sin(4t) \end{cases}$. What is the speed of the particle when $t = 3$?

a $-8\sin 12$ b $\frac{4\cos 12}{6}$ c $\sqrt{(4\cos 12)^2 + 36}$ d $\sqrt{(\sin 12)^2 + 81}$ e $(4\cos 12)^2 + 36$



5 Which of the following integrals represents the area shaded in the graph shown at right? The curve is given by $r = 4\sin 2\theta$.

a $\int_{3\pi/2}^{2\pi} 2\sin(2\theta)\,d\theta$ b $\int_{\pi/2}^{\pi} 8\sin^2(2\theta)\,d\theta$ c $\int_{0}^{\pi} 2\sin^2(2\theta)\,d\theta$

d $\int_{\pi/2}^{\pi} 2\sin(2\theta)\,d\theta$ e $\int_{3\pi/2}^{2\pi} 4\sin^2(2\theta)\,d\theta$

6 Which of the following integrals represents the arc length of the polar function $r = 1 + \cos\theta$ from $0 \le \theta \le \pi$?

a $\int_0^{\pi} \sqrt{(1+\cos\theta)^2 + (-\sin\theta)^2}\,d\theta$ b $\int_0^{\pi} \sqrt{1+\sin^2\theta}\,d\theta$

c $\int_0^{\pi} (1+\cos\theta)\,d\theta$ d $\int_0^{\pi} \frac{1}{2}(1+\cos\theta)^2\,d\theta$

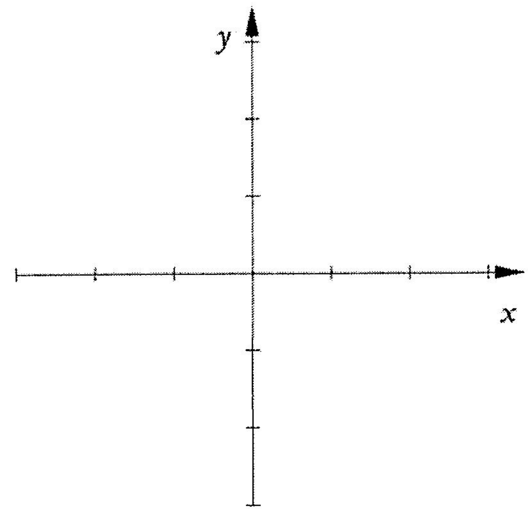
e $\int_0^{\pi} 2\pi(1+\cos\theta)\sin\theta\sqrt{(1+\cos\theta)^2 + (-\sin\theta)^2}\,d\theta$

7 Consider the graph of the vector function $\mathbf{r}(t)=\langle 1+t^3, 3+4t\rangle$. What is the value of $\frac{d^2y}{dx^2}$ at the point on the graph where $x=2$?

a 0 b $\frac{4}{3}$ c $-\frac{8}{3}$ d $-\frac{8}{9}$ e $-\frac{1}{18}$

8 A particle moves so that at time $t>0$ its position vector is $\langle \ln(t^2+2t), 2t^2\rangle$. At time $t=2$, its velocity vector is

a $\langle \frac{3}{4}, 8\rangle$ b $\langle \frac{3}{4}, 4\rangle$ c $\langle \frac{1}{8}, 8\rangle$ d $\langle \frac{1}{8}, 4\rangle$ e $\langle -\frac{5}{16}, 4\rangle$



9 Consider the curves $r_1=2\cos\theta$ and $r_2=\sqrt{3}$.

a Sketch the curves on the axes provided at right.

b Show use of calculus to find the area of the region common to both graphs.

10 Consider the curve given parametrically by $\begin{cases} x=2t^3-3t^2 \\ y=t^3-12t \end{cases}$.

a In terms of t, find $\frac{dy}{dx}$.

b Write an equation for the line tangent to the curve at the point at which $t=-1$.

c Find the x- and y-coordinates for each critical point on the curve and identify each point as having a vertical or horizontal tangent.

11. The asymptotes of the graph of the parametric equations $x=\frac{1}{t}$ and $y=\frac{t}{1+t}$ are

(Hint: rewrite the curve in rectangular coordinates, then find its asymptotes.)

(A) $x=0,\ y=0$ (B) $x=0$ only (C) $x=-1,\ y=0$

(D) $x=-1$ only (E) $x=0,\ y=1$

CALCULATOR-ACTIVE

12 An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t\geq 0$ with $\frac{dx}{dt}=12t-3t^2$ and $\frac{dy}{dt}=\ln\left(1+(t-4)^4\right)$. At time $t=0$, the object is at position $(-13,5)$. At time $t=2$, the object is at point P with x-coordinate 3.

a Find the acceleration vector and the speed at time $t=2$.

b Find the y-coordinate of point P.

c Write an equation for the line tangent to the curve at point P.

d For what value(s) of t, if any, is the object at rest? Justify your answer.