(5.5) --- Law of Sines & (5.6) --- Law of Cosines

Find the area of each triangle to the nearest tenth

1) $a = 5, b = 12, c = 13$

2) $c = 3.58, b = 6.8, A = 39°$

Solve each triangle (round to the nearest tenth)

3) $b = 40, c = 45, A = 51°$

4) $c = 125, b = 150, C = 25°$

5) $a = 20,\ b = 28,\ A = 73°$

6) $a = 18,\ b = 21,\ A = 47°$

7) $a = 17, b = 12, c = 16$

8) Aliens are on their way to earth to abduct Mr. Shahin and Ms. Borchert in order to study brilliant Earthlings. Mr. Shahin looks due East and sees the UFO with an angle of elevation of 40°. At the same time Ms. Borchert is 1 mile due West of Mr. Shahin. When Ms. Borchert looks due East she sees the same UFO at an angle of elevation of 25°. Find the distance between Mr. Shahin and the UFO. How far is the UFO above the ground?

9) As Danielle stands on a bridge she notices that it is supported by triangular braces. The sides of each brace have lengths 63 ft, 46 ft, and 40 ft. In order to keep the bridge from collapsing she needs to find the angle measure opposite the 46 ft side. Help Danielle save the bridge!

10) Mr. Simpson and Mr. Baker walk from opposite ends of a city block to a point on the other side of the street where they are having a *Star Trek* convention. The angle formed by their paths is $25°$. Mr. Baker walks 300 ft, while Mr. Simpson walks 320 ft. How long is the city block?

11) Eric's mom will be serving *Bagel Bites* to Eric's very productive study group when tey arrive. She will be serving them on a new triangular servering platter that Eric gave her for Mother's Day. If one side of the platter is 15 in long and the other two sides both measure 18 inches, find the area of the platter.

12) Ms. Borchert's 3rd period class decided to make a poster to hang on the wall of the classroom in order to declare their superiority over 4th period. To honor their Pre-Calculus knowledge they made a triangular shaped poster. Ms. Borchert's 4th period class wants to make an even bigger poster that cover more wall space. To find the area of the 3rd period's poster they measure and find two of the sides are 8 ft and 9 ft, while the included angle measures $39°$. How large will 4th period's poster have to be in order to cover more area than 3rd periods?

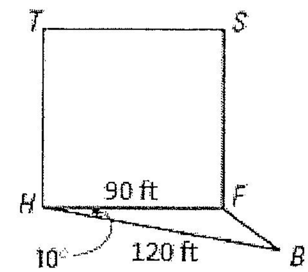
13) The measures of two sides of a parallelogram are 28 in and 42 in. If the longer diagonal has measure 58 in, find the measure of the angles at the vertices.

Name: ______________________

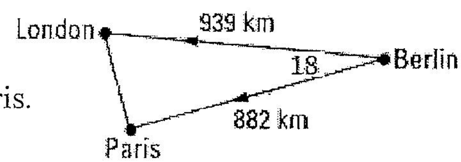
More Practice with Law of Sines and Law of Cosines

Find each indicated side length or angle measure.

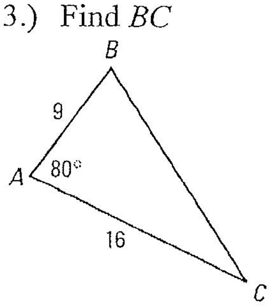
1.) A baseball infield is determined by a square with sides 90 ft long. In the diagram, home plate is *H* and first base is *F*. Suppose the first baseman ran in a straight line from *F* to catch a pop-up at *B*, 120 ft from home plate. If the measure of $\angle FHB$ is 10°, how far did the first baseman run?



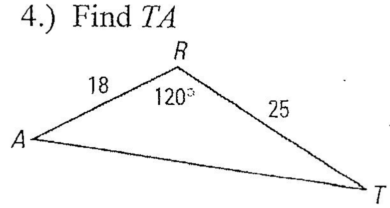
2.) Two airplanes leave Berlin, one heading straight for London and the other straight for Paris. The angle formed is 18 degrees. Use the Law of Cosines to estimate the distance from London to Paris.



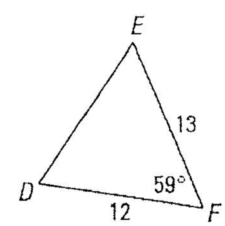
3.) Find *BC*



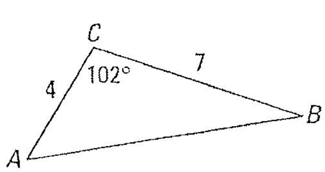
4.) Find *TA*



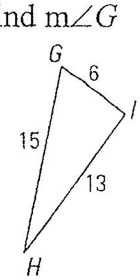
5.) Find DE



6.) Find *AB*



7.) Find $m\angle G$



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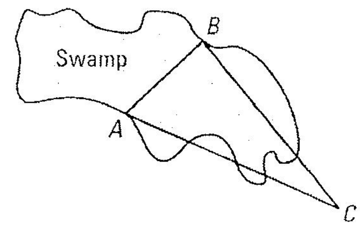
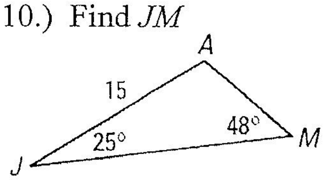
8.) Some students in Geometry are assigned the task of measuring the distance between two trees separated by a swamp. The students determine that the angle formed by tree A, a dry point C, and tree B is 27°. They also know that $m\angle ABC$ is 85°. If AC is 150 ft, how far apart are the trees?

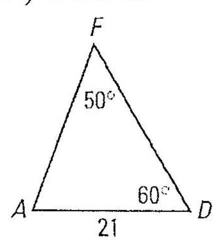


9.) Two lookout towers, L and M, are 50 kilometers apart. The ranger in Tower L sees a fire at point C such that $m\angle CLM = 40°$. The ranger in Tower M sees the same fire such that $m\angle CML = 65°$. How far is the fire from Tower L?

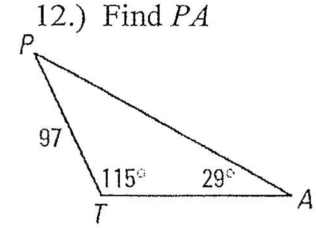
10.) Find JM



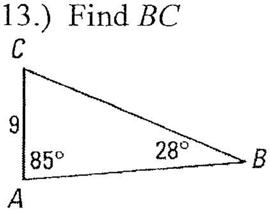
11.) Find DF



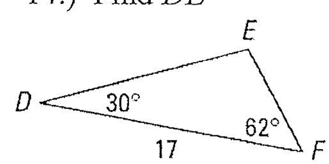
12.) Find PA



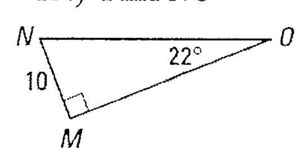
13.) Find BC



14.) Find DE



15.) Find NO



Area of Oblique Triangle Worksheet

Find the **area** of each triangle to the nearest thousandth.

1. $A = 60^0$, $B = 75^0$, $a = 2$

2. $a = 174$, $b = 138$, $c = 188$

3. $B = 60^0$, $C = 75^0$, $a = 8$

4. $a = 11$, $c = 5$, $B = 50^0$

5. $a = 17$, $b = 13$, $c = 19$

6. $A = 62^0$, $b = 146.2$, $c = 209.3$

7. $a = 19.42$, $c = 19.42$, $B = 31^0$

8. $b = 14$, $C = 110^0$, $B = 25^0$

9. The adjacent sides of a parallelogram measure 8 cm and 12 cm, and one angle measures 60^0. Find the area of the parallelogram.

Practice Section 5.1: Simplifying Trigonometric Expressions

Use basic trigonometric identities to simplify. Show all substitutions and algebra.

1. $(\tan\theta)(\cos\theta)$

2. $\csc x - \cos x \cot x$

3. $(\tan\theta)(\cot\theta)$

4. $\sin x + \sin x \cot^2 x$

5. $\cos^2\theta + \sin^2\theta$

6. $(\sin\theta - 1)(\sin\theta + 1)$

7. $(\csc\theta - 1)(\csc\theta + 1)$

8. $\cos\theta\,(\sec\theta - \cos\theta)$

9. $(\cot\theta)(\sec\theta)(\sin\theta)$

10. $(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$

11. $\dfrac{\sin\theta\cos\theta}{1-\cos^2\theta}$

12. $\dfrac{1}{\sec^2\theta} + \dfrac{1}{\csc^2\theta}$

13. $\dfrac{\tan\theta\cos\theta}{\sin\theta}$

14. $\cos^2\theta + (\tan^2\theta)(\cos^2\theta)$

15. $1 + (\csc^2\theta)(\cos^2\theta)$

16. $\sin\theta\csc(-\theta)$

17. $\sec\theta\sin\left(\dfrac{\pi}{2} - \theta\right)$

18. $\sec^2(-\theta) - \tan^2\theta$

Precalculus Name: ______________________

Simplifying Trigonometric Expressions

Simplify each of the following.

1. $\sec x \cos x$

2. $\dfrac{\sin(-x)}{\cos(-x)}$

3. $\tan^2 x - \sec^2 x$

4. $\dfrac{1-\cos^2 x}{\sin x}$

5. $\cot x \sin x$

6. $\dfrac{\sin\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right)}$

7. $\sin x \sec x$

8. $\cos^2 x\left(\sec^2 x - 1\right)$

9. $\dfrac{\sec^2 x - 1}{\sin^2 x}$

10. $\cot x \sec x$

11. $\sec^4 x - \tan^4 x$

12. $\dfrac{\cos^2\left(\frac{\pi}{2}-x\right)}{\cos x}$

13. $\tan\theta \csc\theta$

14. $\sin\theta\left(\csc\theta - \sin\theta\right)$

15. $\cos\beta \tan\beta$

16. $\sec\alpha \dfrac{\sin\alpha}{\tan\alpha}$

17. $\dfrac{\cot x}{\csc x}$

18. $\dfrac{\csc\theta}{\sec\theta}$

19. $\sec^2 x\left(1-\sin^2 x\right)$

20. $\dfrac{1}{\tan^2 x + 1}$

21. $\dfrac{\sin(-x)}{\cos x}$

22. $\dfrac{\tan^2 x}{\sec^2 x}$

23. $\cos\left(\dfrac{\pi}{2}-x\right)\sec x$

24. $\cot\left(\dfrac{\pi}{2}-x\right)\cos x$

25. $\dfrac{\cos^2 y}{1-\sin y}$

26. $\cos t\left(1+\tan^2 t\right)$

27. $\tan^2 x-\tan^2 x\sin^2 x$

28. $\sec^2 x\tan^2 x+\sec^2 x$

29. $\sec^2 x\sin^2 x-\sin^2 x$

30. $\dfrac{\sec^2 x-1}{\sec x-1}$

31. $\tan^4 x+2\tan^2 x+1$

32. $1-2\cos^2 x+\cos^4 x$

33. $\sin^4 x-\cos^4 x$

34. $\csc^3 x-\csc^2 x-\csc x+1$

Perform the indicated operation and simplify.

35. $(\sin x+\cos x)^2$

36. $(\cot x+\csc x)(\cot x-\csc x)$

37. $(\sec x-1)(\sec x+1)$

38. $(3-3\sin x)(3+3\sin x)$

39. $\dfrac{1}{1+\cos x}+\dfrac{1}{1-\cos x}$

40. $\dfrac{1}{\sec x+1}-\dfrac{1}{\sec x-1}$

41. $\dfrac{\cos x}{1+\sin x}+\dfrac{1+\sin x}{\cos x}$

42. $\tan x-\dfrac{\sec^2 x}{\tan x}$

Precalculus Name: ______________________

Proving Trigonometric Identities

1. Prove: $\frac{\csc x}{\sin x} - \frac{\cot x}{\tan x} = 1$

2. Prove: $\frac{\sec^2 x}{\cot x} - \tan^3 x = \tan x$

3. Prove: $\frac{\cot x}{\csc^2 x - 1} = \tan x$

4. Prove: $\tan^2 x \cos^2 x + \cot^2 x \sin^2 x = 1$

5. Prove: $\frac{\sec x - \cos x}{\tan x} = \sin x$

6. Prove: $\frac{1 + \tan x}{\sin x} - \sec x = \csc x$

7. Prove: $\sin x\left(\frac{\sin x}{1 - \cos x} + \frac{1 - \cos x}{\sin x}\right) = 2$

8. Prove: $\sec x \csc^2 x - \csc^2 x = \frac{\sec x}{1 + \cos x}$

9. Prove: $(\sec x - \tan x)^2 = \frac{1 - \sin x}{1 + \sin x}$

10. Prove: $\frac{1 + \sin x}{\cos x \sin x} = \sec x(\csc x + 1)$

11. Prove: $(\sin x - \cos x)^2 + 2\sin x - \cos x = (1 + 2\sin x)(1 - \cos x)$

12. Prove: $\frac{\sin x}{1 + \cos x} + \frac{\cos x}{\sin x} = \frac{1}{\sin x}$

Name ____________________

Date ______________ Pd ______

5.2: Verifying Trigonometric Identities

Verify the identity.

4. $\dfrac{\sin^2 t}{\tan^2 t} = \cos^2 t$

10. $\cot^2 y(\sec^2 y - 1) = 1$

6. $\cos^2 \beta - \sin^2 \beta = 2\cos^2 \beta - 1$

22. $\sec^6 x(\sec x \tan x) - \sec^4 x(\sec x \tan x) = \sec^5 x \tan^3 x$

9. $(1 + \sin x)(1 - \sin x) = \cos^2 x$

24. $\dfrac{\cos[(\pi/2) - x]}{\sin[(\pi/2) - x]} = \tan x$

36. $\sec^2 y - \cot^2\left(\frac{\pi}{2} - y\right) = 1$

63. $\tan^5 x = \tan^3 x \sec^2 x - \tan^3 x$

42. $\frac{1+\csc\theta}{\sec\theta} - \cot\theta = \cos\theta$

65. $\cos^3 x \sin^2 x = (\sin^2 x - \sin^4 x)\cos x$

48. $\frac{\cot\alpha}{\csc\alpha - 1} = \frac{\csc\alpha + 1}{\cot\alpha}$

SOLVING TRIG EQUATIONS

FIRST DEGREE EQUATIONS:

1a) $2x - 1 = 0$; solve for x.

b) $2\cos\theta - 1 = 0$; solve for θ, $0 \leq \theta \leq 2\pi$

2a) $2x + \sqrt{3} = x$; solve for x.

b) $2\tan\theta + \sqrt{3} = \tan\theta$; solve for θ, $0 \leq \theta \leq 2\pi$

3. $3(\csc\theta - 1) = \csc\theta + 2$; solve for θ, $0 \leq \theta \leq 360°$

4. $3\sin\theta + 5 = \sin\theta$; solve for θ, $0 \leq \theta \leq 2\pi$

SECOND DEGREE TRIG EQUATIONS

Examples:

1.a) $x^2 - 3x - 4 = 0$; solve for x

b) $\tan^2\theta - 3\tan\theta - 4 = 0$; solve for θ, $0 \leq \theta \leq 2\pi$

2.a) $x^2 - x = 0$; solve for x

b) $\sin^2\theta - \sin\theta = 0$; solve for θ, $0 \leq \theta \leq 2\pi$

3. $\cos^2\theta - \cos\theta - 6 = 0$; solve for θ, $0 \leq \theta \leq 2\pi$

4. $\tan\theta = \dfrac{1}{\tan\theta}$; solve for θ, $0 \leq \theta \leq 2\pi$

SOLVING TRIG EQUATIONS – USING IDENTITIES: SUBSTITUTE!!!!

Solve for x:

1. $\cos 2x + 2 = \sin x, \quad 0 \le \theta \le 2\pi$

2. $2\cos^2 x - \sin x = 1, \quad 0 \le \theta \le 2\pi$

3. $\sin 2x - \sin x = 0, \quad 0 \le \theta \le 2\pi$

4. $\tan x - 10\cot x = 3, \quad 0 \le \theta \le 2\pi$

5. $\cos \frac{1}{2} x = 1, \quad 0 \le \theta \le 2\pi$

Practice Solving Trigonometric Equations with Multiple Angles

Solve each equations on the interval $[0, 2\pi)$

25. $\sin 2x = \dfrac{\sqrt{3}}{2}$

26. $\cos 2x = \dfrac{\sqrt{2}}{2}$

27. $\cos 4x = -\dfrac{\sqrt{3}}{2}$

28. $\sin 4x = -\dfrac{\sqrt{2}}{2}$

29. $\tan 3x = \dfrac{\sqrt{3}}{3}$

30. $\tan 3x = \sqrt{3}$

31. $\tan \dfrac{x}{2} = \sqrt{3}$

32. $\tan \dfrac{x}{2} = \dfrac{\sqrt{3}}{3}$

33. $\sin \dfrac{2\theta}{3} = -1$

34. $\cos \dfrac{2\theta}{3} = -1$

35. $\sec \dfrac{3\theta}{2} = -2$

36. $\cot \dfrac{3\theta}{2} = -\sqrt{3}$

Precalculus

Name:

Solving Trigonometric Equations

1. $2\cos x + 1 = 0$

2. $2\sin x - 1 = 0$

3. $\sqrt{3}\csc x - 2 = 0$

4. $\tan x + 1 = 0$

5. $2\sin^2 x = 1$

6. $\tan^2 x = 3$

7. $3\sec^2 x - 4 = 0$

8. $\csc^2 x - 2 = 0$

9. $\tan x(\tan x - 1) = 0$

10. $\cos x(2\cos x + 1) = 0$

11. $\sin x(\sin x+1)=0$

12. $4\sin^2 x-3=0$

13. $\sin^2 x=3\cos^2 x$

14. $(3\tan^2 x-1)(\tan^2 x-3)=0$

15. $\sec x\csc x-2\csc x=0$

16. $\sec^2 x-\sec x-2=0$

17. $2\sin^2 x+3\sin x+1=0$

18. $3\tan^3 x-\tan x=0$

19. $2\sec^2 x+\tan^2 x-3=0$

20. $2\sin^2 x=2+\cos x$

21. $2\sin x+\csc x=0$

22. $\csc x+\cot x=1$

Precalculus

Name: ____________________

Sum and Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Use the sum or difference identity to find the exact value.

1. $\sin 15^o$

2. $\tan 15^o$

3. $\sin 75^o$

4. $\cos 75^o$

5. $\cos \frac{\pi}{12}$

6. $\sin \frac{7\pi}{12}$

7. $\tan \frac{5\pi}{12}$

8. $\tan \frac{11\pi}{12}$

9. $\cos \frac{7\pi}{12}$

10. $\sin \frac{-\pi}{12}$

Write the expression as the sine, cosine, or tangent of an angle

11. $\sin 42^{o} \cos 17^{o} - \cos 42^{o} \sin 17^{o}$

12. $\cos 94^{o} \cos 18^{o} + \sin 94^{o} \sin 18^{o}$

13. $\sin \frac{\pi}{5} \cos \frac{\pi}{2} + \sin \frac{\pi}{2} \cos \frac{\pi}{5}$

14. $\sin \frac{\pi}{3} \cos \frac{\pi}{7} - \cos \frac{\pi}{3} \sin \frac{\pi}{7}$

15. $\dfrac{\tan 19^{o} + \tan 47^{o}}{1 - \tan 19^{o} \tan 47^{o}}$

16. $\dfrac{\tan \frac{\pi}{5} - \tan \frac{\pi}{3}}{1 + \tan \frac{\pi}{5} \tan \frac{\pi}{3}}$

17. $\cos x \cos \frac{\pi}{7} + \sin x \sin \frac{\pi}{7}$

18. $\cos x \cos \frac{\pi}{7} - \sin x \sin \frac{\pi}{7}$

19. $\sin 3x \cos x - \cos 3x \sin x$

20. $\cos 7y \cos 3y - \sin 7y \sin 3y$

21. $\dfrac{\tan 2y + \tan 3x}{1 - \tan 2y \bullet \tan 3x}$

22. $\dfrac{\tan 3\alpha - \tan 2\beta}{1 + \tan 3\alpha \bullet \tan 2\beta}$

14-6 Skills Practice

Double-Angle and Half-Angle Formulas

Find the exact values of $\sin 2\theta$, $\cos 2\theta$, $\sin \frac{\theta}{2}$, and $\cos \frac{\theta}{2}$ for each of the following.

1. $\cos \theta = \frac{7}{25}$, $0° < \theta < 90°$

2. $\sin \theta = -\frac{4}{5}$, $180° < \theta < 270°$

3. $\sin \theta = \frac{40}{41}$, $90° < \theta < 180°$

4. $\cos \theta = \frac{3}{7}$, $270° < \theta < 360°$

5. $\cos \theta = -\frac{3}{5}$, $90° < \theta < 180°$

6. $\sin \theta = \frac{5}{13}$, $0° < \theta < 90°$

Find the exact value of each expression by using the half-angle formulas.

7. $\cos 22\frac{1}{2}°$

8. $\sin 165°$

9. $\cos 105°$

10. $\sin \frac{\pi}{8}$

11. $\sin \frac{15\pi}{8}$

12. $\cos 75°$

Verify that each of the following is an identity.

13. $\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$

14. $\tan \theta + \cot \theta = 2 \csc 2\theta$

NAME ______________________ DATE __________ PERIOD _____

14-6 Practice

Double-Angle and Half-Angle Formulas

Find the exact values of $\sin 2\theta$, $\cos 2\theta$, $\sin \frac{\theta}{2}$, and $\cos \frac{\theta}{2}$ for each of the following.

1. $\cos \theta = \frac{5}{13}$, $0° < \theta < 90°$

2. $\sin \theta = \frac{8}{17}$, $90° < \theta < 180°$

3. $\cos \theta = \frac{1}{4}$, $270° < \theta < 360°$

4. $\sin \theta = -\frac{2}{3}$, $180° < \theta < 270°$

Find the exact value of each expression by using the half-angle formulas.

5. $\tan 105°$

6. $\tan 15°$

7. $\cos 67.5°$

8. $\sin\left(-\frac{\pi}{8}\right)$

Verify that each of the following is an identity.

9. $\sin^2 \frac{\theta}{2} = \frac{\tan \theta - \sin \theta}{2 \tan \theta}$

10. $\sin 4\theta = 4 \cos 2\theta \sin \theta \cos \theta$

11. AERIAL PHOTOGRAPHY In aerial photography, there is a reduction in film exposure for any point X not directly below the camera. The reduction E_θ is given by $E_\theta = E_0 \cos^4 \theta$, where θ is the angle between the perpendicular line from the camera to the ground and the line from the camera to point X, and E_0 is the exposure for the point directly below the camera. Using the identity $2 \sin^2 \theta = 1 - \cos 2\theta$, verify that $E_0 \cos^4 \theta = E_0\left(\frac{1}{2} + \frac{\cos 2\theta}{2}\right)^2$.

12. IMAGING A scanner takes thermal images from altitudes of 300 to 12,000 meters. The width W of the swath covered by the image is given by $W = 2H' \tan \theta$, where H' is the height and θ is half the scanner's field of view. Verify that $\frac{2H' \sin 2\theta}{1 + \cos 2\theta} = 2H' \tan \theta$.

Double Angle Identities + Sum & Difference

Find the exact value of the expression:

1. $\sin 15^\circ$

2. $\cos -112.5^\circ$

3. $\tan(\pi/12)$

4. $\sin(\pi/8)$

5. $\sin 22.5^\circ$

6. $\sin(165^\circ)$

7. $\cos(75^\circ)$

8. $\tan(105^\circ)$

Find the exact values of $\sin\frac{u}{2}$, $\cos\frac{u}{2}$, and $\tan\frac{u}{2}$.

9. $\cos u = \frac{4}{5}$, $\frac{3\pi}{2} < u < 2\pi$

10. $\sin u = \frac{7}{25}$, $\frac{\pi}{2} < u < \pi$

11. $\cos u = \frac{7}{25}$, $0^\circ < u < 90^\circ$

12. $\sin u = \frac{-4}{5}$, $180^\circ < u < 270^\circ$

Find the exact values of sin(2x), cos(2x), and tan (2x).

13. $\sin x = \frac{4}{5}, \quad 0 < x < \frac{\pi}{2}$

14. $\cos x = -\frac{1}{3}, \quad \frac{\pi}{2} < x < \pi$

15. $\sin u = \frac{40}{41}, \quad 90° < u < 180°$

16. $\cos u = \frac{3}{7}, \quad 270° < u < 360°$

Rewrite the expression without double angles or half angles, given that $0 < x < \frac{\pi}{2}$. **Then simplify.**

17. $\sin 4x$

18. $\frac{\cos 2x}{\cos x}$

19. $\cos 2x + \sin x$

Verify they following identities:

20. $\sin 10x = 2\sin 5x \cos 5x$

21. $\cos^2 3x - \sin^2 3x = \cos 6x$

22. $\sin 3x = \sin x(3 - 4\sin^2 x)$

23. $sin2\theta = \frac{2tan\epsilon}{1+tan^2\theta}$ **24.** $tan\theta + cot\theta = 2csc2\theta$ **25.** $sin4\theta = 4cos2\theta sin\theta cos0$

Solve the equation over the interval [0, 2π):

26. $\cos 2x = -\sin x$ **27.** $\cos 2x + \cos x = 0$ **28.** $\sin 2x + \sqrt{2} \sin x = 0$

Evaluate the expression given sin$u = \frac{5}{13}$ with $\pi < u < \frac{3\pi}{2}$ and $v = \frac{\pi}{3}$

29. $\cos (u + v)$ **30.** $\tan (u - v)$

Simplify the expressions below (and for #23 & #24 EVALUATE)

31. $\sin (x + \pi/2)$ **32.** $\tan (x - \pi)$ **33.** $-\sin 40°\cos32° + \sin32°\cos40°$

34. $-\cos\frac{5\pi}{3}\cos\frac{\pi}{3}+\sin\frac{\pi}{3}\sin\frac{5\pi}{3}$

35. $\frac{\tan 135° - \tan 15°}{1+\tan 135° \tan 15°}$

Solve the equation for $0 \le x < 2\pi$

36. $\sin\left(x+\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{3}\right)=1$

37. $\cos\left(x+\frac{\pi}{4}\right)+\cos\left(x-\frac{\pi}{4}\right)=1$