Pre-Calculus Name: _____

Chapter 6 Review --- Vectors, Polars, & Parametrics

In the following exercises, eliminate the parameter and describe the graph of the function. (Be sure you take domain and range of the parametric into account)

1. $x = t$ & $y = 2t - 1$

2. $x = 4\cos^2\theta$ & $y = 2\sin\theta$

3. $x = 4t - 1$ & $y = 2t + 3$

4. $x = t + 3$ & $y = t^2$

5. $x = \sqrt[3]{t}$ & $y = 3 - t^2$

6. $x = t^2 - 1$ & $y = t^2 + 1$

7. $x = t - 2$ & $y = \dfrac{t}{t-2}$

9. $x = |t - 3|$ & $y = t + 3$

10. $x = \sec^2\theta$ & $y = \tan^2\theta$

11. $x = \cos\theta$ & $y = 4\sin\theta$

12. $x = e^t$ & $y = e^{-t}$

13. $x = t^5$ & $y = 5\ln t$

14. A dart is thrown upward from 6 ft. high with an initial velocity of 18 feet/sec at an angle of elevation of 41°

a. Write a parameterization describing the position of the dart at time t.

$x(t) =$ _____ $y(t) =$ _____

b. Approximately how long will it take for the dart to hit the ground?

c. Find the approximate maximum height of the dart.

d. How long will it take for the dart to reach maximum height?

15. An arrow is shot from a platform 20 feet off the ground with an initial velocity of 150 feet/sec at an angle of elevation of 23°.

a. Write a parameterization describing the position of the arrow at time t.

$x(t) =$ _____ $y(t) =$ _____

b. Find the approximate maximum height of the arrow.

c. Approximately how long will it take for the arrow to reach maximum height?

d. There is a wall 30 feet high 500 feet from the archer. Will the arrow hit it? If so, how long will it take to hit it?

16. A golfer hits a ball with an initial velocity of 90 mph at angle of elevation of 64°.

a, Write a parametric equation that describes the position of the ball at time t.

$x(t) =$ _____ $y(t) =$ _____

b. Approximately how long will it take for the ball to hit the ground?

c. Find the approximate maximum height of the ball.

d, The green is 150 yards away. Will the ball reach the green? Explain.

17. An NFL kicker at the 33-yard line attempts a field goal. The ball leaves his foot at 69 feet/sec at an angle of elevation of 38°.

a. Write a parametric equation that describes the position of the ball at time t.

$x(t) =$ _____ $y(t) =$ _____

b. How high does the ball get above the field?

c. The goal posts are 10 ft high & 45 yards away from him. If the kick is straight, is the field goal good? Explain

18. Jack and Jill are standing 60 feet apart. At the same time, they each throw a softball from an initial height of two feet towards each other. Jack throws the softball at an initial velocity of 45 ft/sec at an angle of elevation of 44°. Jill throws her ball with an initial velocity of 41 ft/sec with an angle of elevation of 37°.

a. Write 2 parameterizations describing the position of the balls at time t.
Remember they are throwing the balls toward each other.

$x_1(t) =$ _____ $y_1(t) =$ _____ $x_2(t) =$ _____ $y_2(t) =$ _____

b. Find the heights of each ball when one is directly above the other.

c. About how far has each ball traveled when one is directly above the other?

d. When does each ball hit the ground?

19. Convert the following polar points to rectangular coordinates.

a. $\left(6,\frac{\pi}{2}\right)$ **b.** $\left(-1,\frac{7\pi}{4}\right)$ **c.** $\left(-4,-\frac{\pi}{3}\right)$

d. $(3, 120°)$ **e.** $(8, 210°)$ **f.** $(10, 72°)$ **Need calc for this one

20. Convert the following rectangular points to polar coordinates

a. $(-3, 3)$ **b.** $\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$ **c.** $(0, -4)$

d. $\left(-5\sqrt{2},-5\sqrt{2}\right)$ **e.** $(-\sqrt{3}, 1)$ **f.** $(7, -24)$

21. For each of the following rectangular equations, change it to polar form.

a. $x^2 - y^2 = 4$ **b.** $xy = 12$ **c.** $5x - y = 7$

d. $(x-1)^2 + y^2 = 1$ **e.** $y = x\sqrt{3}$ **f.** $x^2 + y^2 + 4x = 0$

22. For each of the following polar equations, change it to rectangular form.

a. $r = 4$ **b.** $\tan^2\theta = 9$ **c.** $r = 8\csc\theta$

d. $r = 8\cos\theta$ **e.** $r = \frac{5}{2\sin\theta - \cos\theta}$ **f.** $r = \frac{1}{1+\cos\theta}$

23. Match the polar equations with their graphs below. (all)

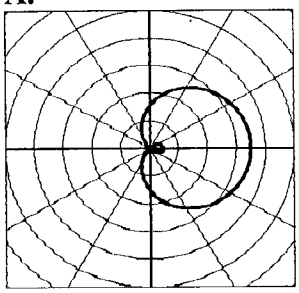
___1) $r = 2.5 + 2.5\sin\theta$ ___2) $r = 3$ ___3) $r = 3.5\sin(3\theta)$ ___4) $r = 4.5\sin(2\theta)$

___5) $r = 4.5\cos(2\theta)$ ___6) $r = 1.5 + 2\cos\theta$ ___7) $r = -3\sin\theta$ ___8) $r = 2 - \sin\theta$

___9) $r^2 = 16\sin(2\theta)$ ___10) $r = 4\cos(5\theta)$ ___11) $r = 3.5\cos(3\theta)$ ___12) $r = 2.5 - 2.5\cos\theta$

___13) $r = 3\cos\theta$ ___14) $r = 1 + 4\sin\theta$ ___15) $r = 4.5\sin(6\theta)$ ___16) $r = \frac{1}{2}\theta$

A.



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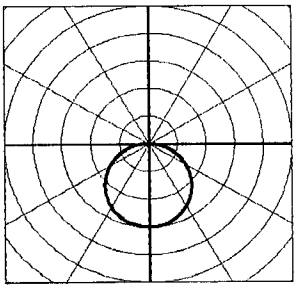

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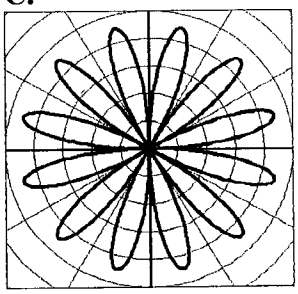

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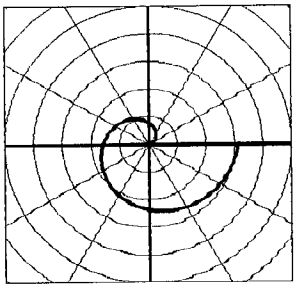

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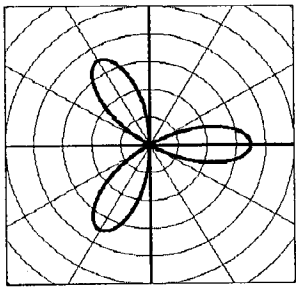

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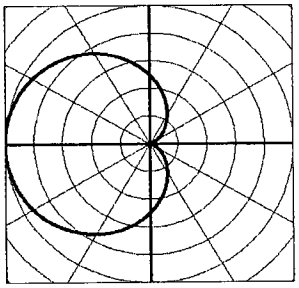

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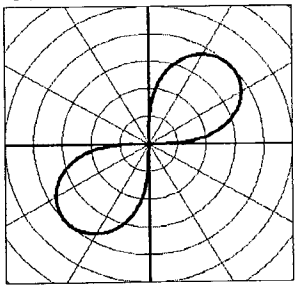

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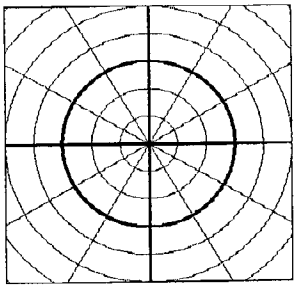

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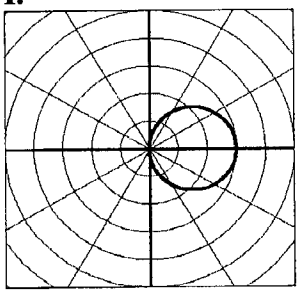

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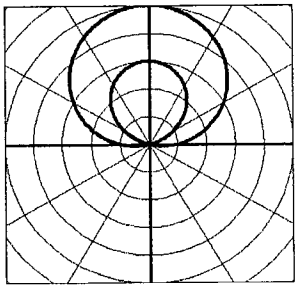

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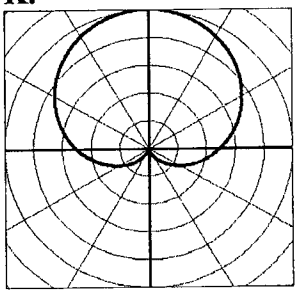

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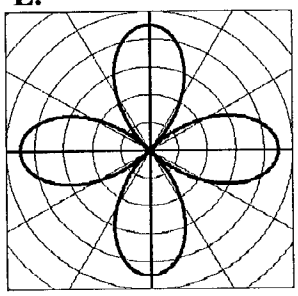

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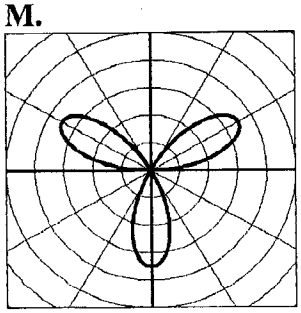

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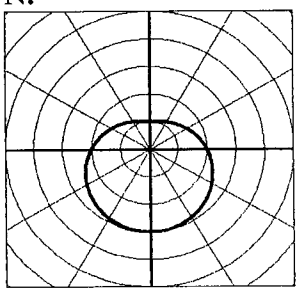

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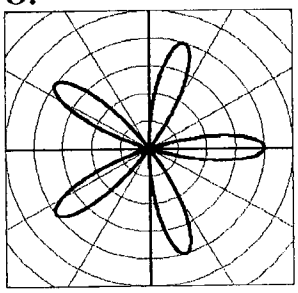

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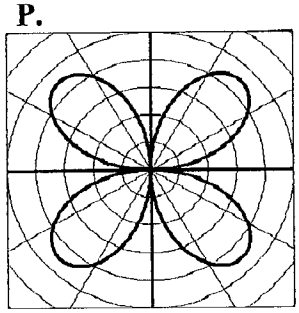

25. Find component form and the magnitude of the vector **v** with initial point P and terminal point Q.

a. $P(-3,7), Q(2,-1)$ **b.** $P(6,-2), Q(-1,-2)$ **c)** $P(-4,3), Q(-5,1)$

$\mathbf{v} =$ _____ $|\mathbf{v}| =$ ___ $\mathbf{v} =$ _____ $|\mathbf{v}| =$ ___ $\mathbf{v} =$ _____ $|\mathbf{v}| =$ ___

26. Given the vectors $\mathbf{u} = \langle 1,-3 \rangle$, $\mathbf{v} = \langle 3,9 \rangle$, find the following:

a. $\mathbf{u} + \mathbf{v}$ **b.** $\mathbf{u} - \mathbf{v}$ **c.** $8\mathbf{u} - 5\mathbf{v}$ **d.** $\mathbf{u} \cdot \mathbf{v}$

e. $\text{proj}_{\mathbf{v}}\mathbf{u}$ **f.** Write **u** as the sum of 2 orthogonal vectors (one of which is $\text{proj}_{\mathbf{v}}\mathbf{u}$) **g.** The angle between **u** and **v**

27. Find a unit vector in the direction of the following vectors and show that it has length 1.

a. $\mathbf{v} = \langle 2,-13 \rangle$ **b.** $\mathbf{v} = \langle -9,-7 \rangle$ **c.** $\mathbf{v} = \langle -14,0 \rangle$

28. Let **u** be the vector with initial point (13, 5) and terminal point (–12, 5) and let $\mathbf{v} = 8\mathbf{i} + 6\mathbf{j}$. Write the following as a linear combination of **i** and **j**.

a. $-2\mathbf{u}$ **b.** $\mathbf{u} - 2\mathbf{v}$ **c.** $\dfrac{\mathbf{u}}{|\mathbf{v}|}$

29. Write the vector **v** given its magnitude and direction angle.

a. $|\mathbf{v}| = 13 \quad \theta = 60°$ **b.** $|\mathbf{v}| = 20 \quad \theta = 150°$ **c.** $|\mathbf{v}| = 5 \quad \theta =$ direction of $24\mathbf{i} - 7\mathbf{j}$

30. A plane is flying on a bearing of 320° at 375 mph. A wind is blowing with the bearing 305° at 45mph.

a. Write a vector (in component form) of the velocity produced by the airplane alone.

p = _____

b. Write a vector (in component form) of the velocity of the wind.

w = _____

c. Write a vector (in component form) of the actual velocity of the plane.

v = _____

d. Find the actual speed and direction angle (not the bearing) of the plane.

speed = _____ θ = _____

31. Find the vector projection **u** onto **v**. Then write **u** as a sum of two orthogonal vectors, one of which is $\text{proj}_{\mathbf{v}}\mathbf{u}$

$\mathbf{u} = \langle 3, -9 \rangle$ & $\mathbf{v} = \langle -1, 7 \rangle$

$\text{proj}_{\mathbf{v}}\mathbf{u}$ = _____

u = _____