

Converting Degrees to Radians and Radians to Degrees

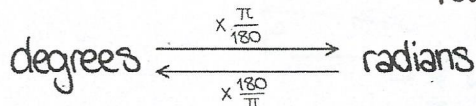
$$C = \pi d = 2\pi r$$

$$2\pi r = 360^\circ$$

$$r = \frac{360^\circ}{2\pi} = \frac{180^\circ}{\pi}$$

$$\frac{2\pi r}{360^\circ} = \frac{360^\circ}{360^\circ}$$

$$\frac{\pi}{180^\circ} r = 1^\circ$$



Example 1 Convert 40° to radians.

$$40^\circ \cdot \frac{\pi}{180^\circ} = \frac{2\pi}{9}$$

Example 2 Convert $\frac{7\pi}{6}$ to degrees.

$$\frac{7\pi}{6} \cdot \frac{180^\circ}{\pi} = 210^\circ$$

Example 3 Use a calculator to convert 16° to radians.

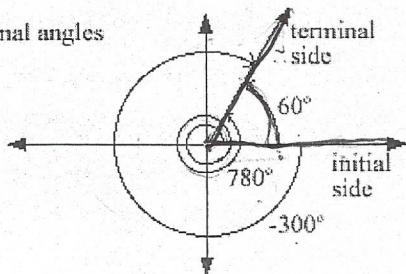
$$16^\circ \cdot \frac{\pi}{180^\circ} = 0.279$$

Example 4 Use a calculator to convert $\frac{3\pi}{8}$ to degrees.

$$\frac{3\pi}{8} \cdot \frac{180}{\pi} = 67.5^\circ$$

coterminal angles are angles in standard position (angles with the initial side on the positive x-axis) that have a common terminal side.

coterminal angles



Degree measure	Radian measure
Formula $\theta + 360n$	Formula $\theta + 2n\pi$
Use positive integer values for n to find the positive coterminals and negative integer values to find negative coterminal angles.	

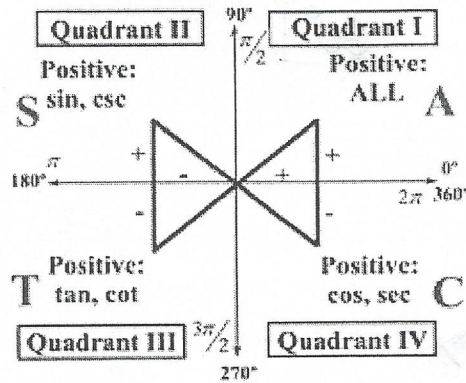
Example 5 Find two other angles, one negative and one positive, which are coterminal to θ .

A. $\theta = 85^\circ$
 445°
 -275°

B. $\theta = -337^\circ$
 23°
 -697°

C. $\theta = \frac{2\pi}{3}$
 $\frac{2}{3}\pi + 2\pi = \frac{8}{3}\pi$
 $\frac{2\pi}{3} - 2\pi = -\frac{4\pi}{3}$

D. $\theta = -3\pi$
 $-\pi$
 π



Example 6 Find the exact value of each of the six trig functions of the given angle.

A. $\theta = \frac{2\pi}{3}$ $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$

$\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

$\cos \frac{2\pi}{3} = -\frac{1}{2}$

$\tan \frac{2\pi}{3} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$

$\csc \frac{2\pi}{3} = \frac{2}{\frac{\sqrt{3}}{\sqrt{3}}} = \frac{2\sqrt{3}}{3}$

$\sec \frac{2\pi}{3} = -2$

$\cot = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}$

B. $\theta = 675^\circ$ $(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$

$\sin 675^\circ = -\frac{\sqrt{2}}{2}$

$\cos 675^\circ = \frac{\sqrt{2}}{2}$

$\tan 675^\circ = -1$

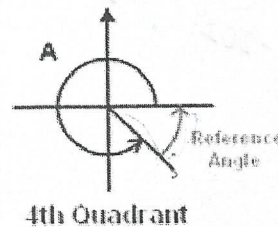
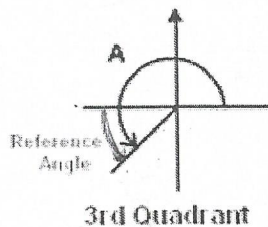
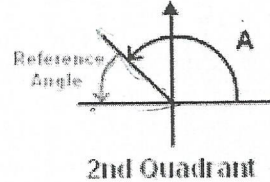
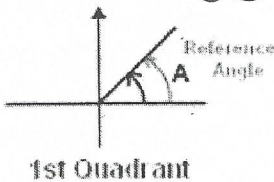
$\csc 675^\circ = -\frac{2}{\frac{\sqrt{2}}{\sqrt{2}}} = -\frac{2\sqrt{2}}{\sqrt{2}} = -\sqrt{2}$

$\sec 675^\circ = \frac{2}{\sqrt{2}} = \sqrt{2}$

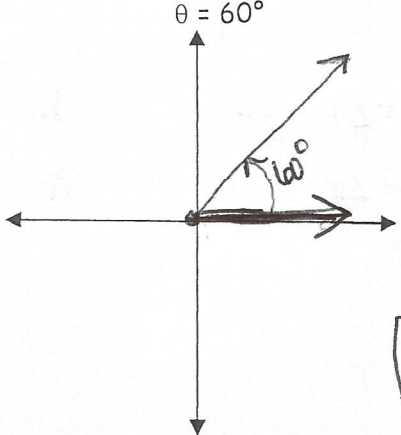
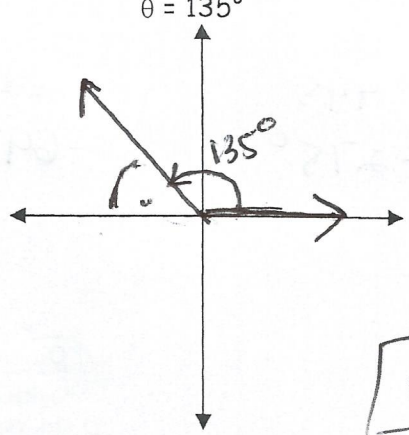
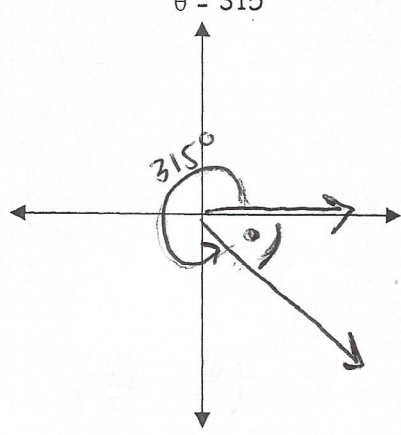
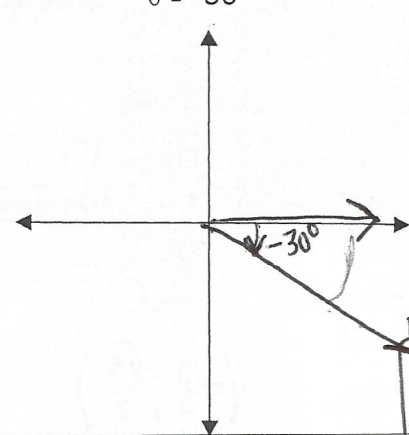
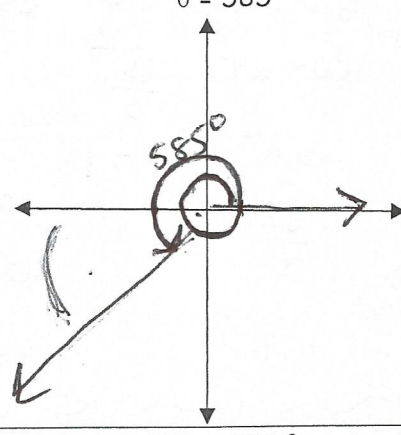
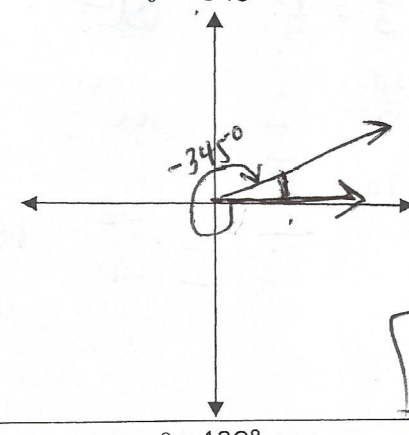
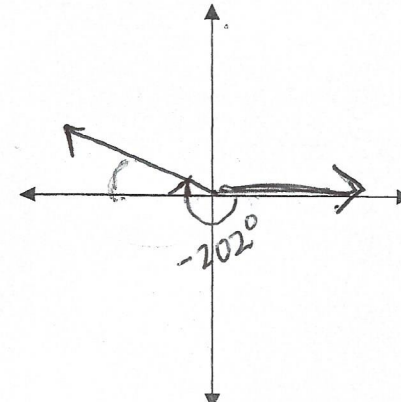
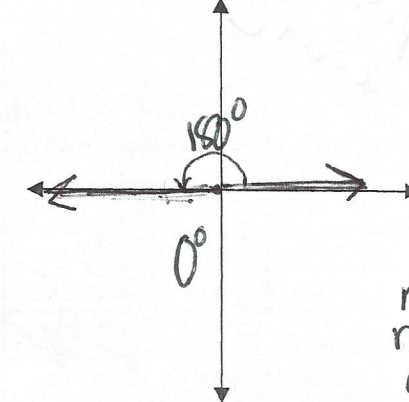
$\cot 675^\circ = -1$

reference angle -- the angle that the given angle makes with the x-axis. Regardless of where the angle ends (that is, regardless of the location of the terminal side of the angle), the reference angle measures the closest distance of that terminal side to the x-axis.

positive acute



Example 7 Draw each angle in standard position, and find its reference angle.

<p>$\theta = 60^\circ$</p>  <p>60°</p>	<p>$\theta = 135^\circ$</p>  <p>45°</p>
<p>$\theta = 315^\circ$</p>  <p>45°</p>	<p>$\theta = -30^\circ$</p>  <p>30°</p>
<p>$\theta = 585^\circ$</p>  <p>45°</p>	<p>$\theta = -345^\circ$</p>  <p>15°</p>
<p>$\theta = -202^\circ$</p>  <p>22°</p>	<p>$\theta = 180^\circ$</p>  <p>no ref. angle</p>