Zeros of Polynomial Functions

Investigation 2 - Local Extrema and Zeros

Zeros of Polynomial Functions

Recall that finding the real-number zeros of a function f is equivalent to finding the x-intercepts of the graph of $y = f(x)$ or the solutions to the equation $f(x) = 0$.

cubic $y=x^3$

a) Investigate various third degree polynomial functions using the graphing calculator to see how many extrema and how many zeros the function can have. Sketch a graph below.

→ at most 3, at least 1

→ at most 2 (1 min, 1 max)

b) Investigate various ~~sixth~~ fourth degree polynomial functions using the graphing calculator to see how many extrema and how many zeros the function can have. Sketch a graph below.

→ at most 4

at most 3

➔ In general, if your degree is "n", how many extrema are possible? $n-1$

➔ In general, if your degree is "n", how many zeros are possible? n

EXAMPLE: Finding the Zeros of a Polynomial Function

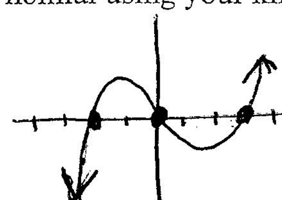
Find the zeros of $f(x) = x^3 - x^2 - 6x$ and then sketch the graph of the polynomial using your knowledge of intercepts and end behavior (so, not a calculator).

$x^3 - x^2 - 6x = 0$

$x(x^2 - x - 6) = 0$

$x^1(x-3)^1(x+2)^1 = 0$

$\underbrace{x=0 \quad x-3=0 \quad x+2=0 \quad x=3 \quad x=-2}_{\text{zeros}}$



REB

$\lim_{x\to\infty} f(x) = \infty$

LEB

$\lim_{x\to-\infty} f(x) = -\infty$

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*******DEFINITION Multiplicity of a Zero of a Polynomial Function*******

If f is a polynomial function and $(x - c)^m$ is a factor of f, then c is a zero of **multiplicity** $\boldsymbol{m}$ of f.

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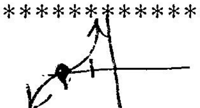
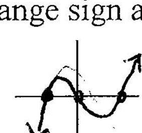
$y = (x+2)^3$ zero $x = -2$ mult $= 3$



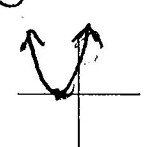
Zeros of Odd and Even Multiplicity

If a polynomial function f has a real zero c of odd multiplicity, then the graph of f crosses the x-axis at $(c, 0)$ and the value of f changes sign at $x = c$. If a polynomial function f has a real zero c of even multiplicity, then the graph of f does not cross the x-axis at $(c, 0)$ and the value of f does not change sign at $x = c$.

So… If the multiplicity of a zero is 1 it will cross the x-axis in the typical "straight through" manner.

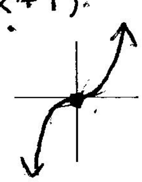


……If the multiplicity of a zero is EVEN it will bounce at the x-axis & will NOT cross through.



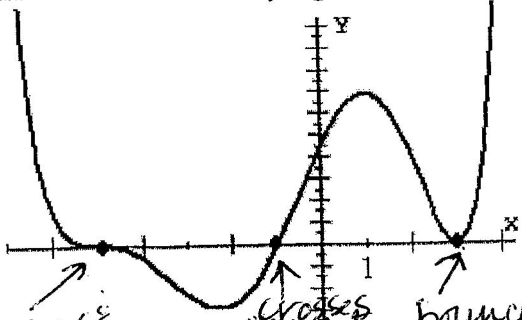
$y = (x+1)^2$

…if the multiplicity of a zero is **GREATER THAN 1 & ODD** it will flatten at the x-axis & WILL cross through.



$y = x^3$

EXAMPLE: Identifying zeros of a polynomial function.



Multiplicity of 1: $x = -1$
cross

Even Multiplicity: $x = 3$
bounce

Odd Multiplicity > 1: $x = -5$
flattens

general eqn. $y = (x+1)(x-3)^2(x+5)^3$

Sketching the Graph of a Factored Polynomial

EXAMPLE: Sketching the Graph of a Factored Polynomial
State the degree and list the zeros of the function. State the multiplicity of each zero and whether the graph crosses the x-axis at the corresponding x-intercept. Then sketch the graph of f by hand.

leading term x^5

(a) $f(x) = (x+2)^3(x-1)^2$

degree: 5

zeros: $x=-2$ mult 3 flatten; $x=1$ mult. 2 bounce

REB → ∞
LEB → −∞

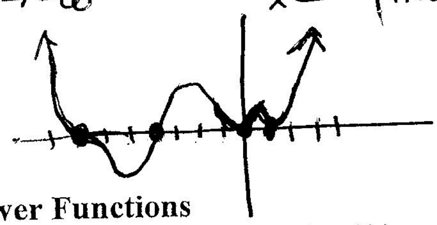
leading term x^{10}

(b) $f(x) = x^2(x+7)^3(x-1)^4(x+4)^1$

degree: 10

zeros: $x=0$ mult. 2 bounce
$x=-7$ mult. 3 flatten
$x=1$ mult 4 bounce
$x=-4$ mult 1 cross.

REB → ∞
LEB → ∞

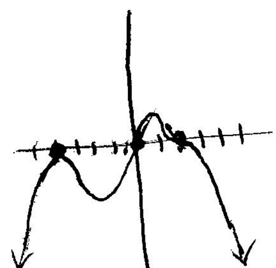


leading term (−x

(c) $f(x) = -x^1(x+4)^2(x-2)^3$

degree: 6

zeros: $x=0$ mult 1
$x=-4$ mult 2
$x=2$ mult 3



REB
LEB

Graphing Transformations of Power Functions
Describe how to transform the graph of an appropriate monomial function $f(x) = a_n x^n$ into the graph of the given function. Sketch the transformed graph by hand compute the location of the y-intercept to check on the graph.

(a) $g(x) = 4(x+1)^3$ **(b)** $h(x) = -(x-2)^4 + 5$ **(c)** $f(x) = -2(x-3)^5 - 1$ **(d)** $k(x) = (x-3)^{2/3} + 2$